

Chromatic Polynomials, Potts Models, and All That

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The chromatic polynomial, introduced by Birkhoff in 1912, counts the number of ways of properly coloring the vertices of a graph G with a given number of available colors. The same polynomial arises in statistical physics, in connection with the Potts model (1952). The complex zeros of these polynomials are of great interest both to combinatorialists and to statistical mechanicians. Indeed, as Yang and Lee showed in 1952, the complex zeros of the partition function give information about phase transitions in the underlying physical system. I begin by giving an introduction to all these problems. I then sketch some recent results and some open problems. This talk is intended to be understandable to both mathematicians and physicists; no prior knowledge of either graph theory or statistical mechanics is required.