Mathematica (11)

In what follows, we use Mathematica to calculate the coefficients of $f(x) = x^2$ which is periodic with period 2π.

First we calculate a_0. Then we calculate the a_n. (Because this function is even, the b_n's are all zero.) Then we can tabulate the first 7 a_n's. We see that these are of the form 4 times $(-1)^n\ 1/n^2$.

```
In[1]:= Integrate[x^2 / (2 Pi), {x, -Pi, Pi}]
```

Out[1]= $\frac{\pi^2}{3}$

```
In[2]:= Element[n, Integers]

Out[2]= n ∈ Integers

In[4]:= Integrate[(x^2 Cos[n x]) / Pi, {x, -Pi, Pi}]
```

Out[4]= $\frac{4\, n\, \pi\, \text{Cos}[n\,\pi] + 2\,(-2 + n^2\,\pi^2)\,\text{Sin}[n\,\pi]}{n^3\,\pi}$

```
In[5]:= Table[%, {n, 1, 7}]
```

Out[5]= $\left\{-4, 1, -\frac{4}{9}, \frac{1}{4}, -\frac{4}{25}, \frac{1}{9}, -\frac{4}{49}\right\}$

```
In[6]:= Table[%%/4, {n, 1, 7}]
```

Out[6]= $\left\{-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}, \frac{1}{36}, -\frac{1}{49}\right\}$

Thus the Fourier series for x^2 is $\pi^2/3 + 4(-\cos(x) + 1/4\cos(2x) - 1/9\cos(3x) + 1/16\cos(4x) - + \ldots)$

It is interesting to apply Parseval's theorem to this Fourier series. First we calculate $f^2(x)$.

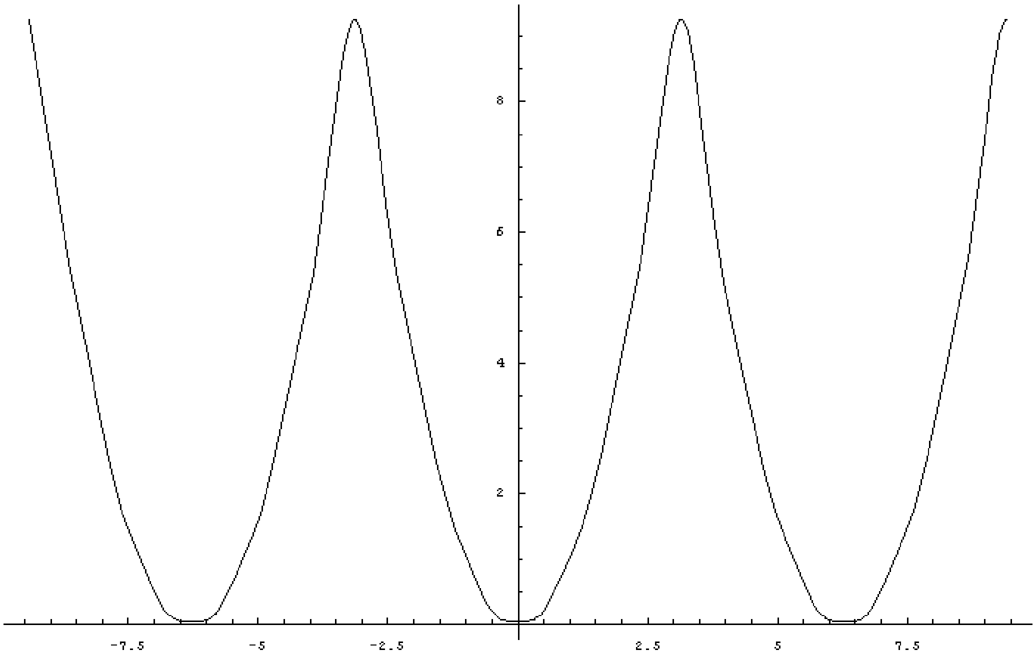
```
In[7]:= Integrate[x^4 / (2 Pi), {x, -Pi, Pi}]
```

Out[7]= $\frac{\pi^4}{5}$

$a_0^2 = \pi^4/9$. The remaining a_n^2 coefficients have the form $4^2(1/n^4)$ and have to be multiplied by the factor ½. Assembling all the factors gives $\pi^4/90 = \Sigma\ 1/n^4$.

A plot of f(x) is given below:

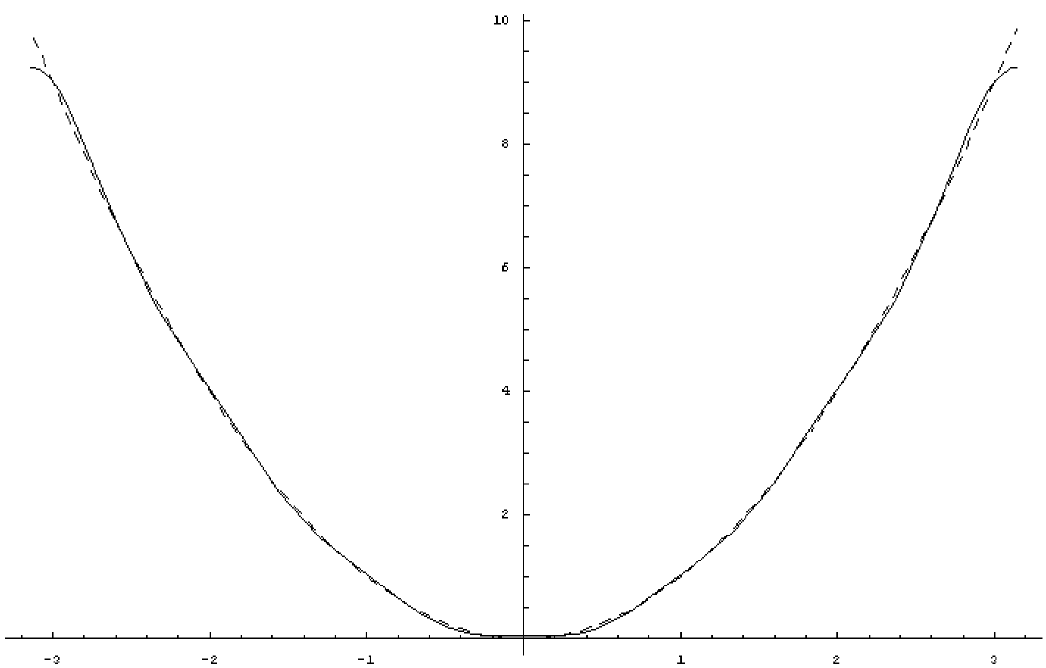
```
f[x_] = Pi^2 / 3 - 4 (Cos[x] - Cos[2 x] / 4 + Cos[3 x] / 9 - Cos[4 x] / 16 + Cos[5 x] / 25 - Cos[6 x] / 36);

Plot[f[x], {x, -3 Pi, 3 Pi}]
```



- Graphics -

A comparison of f(x) and x^2 over one period is shown below:

```
Plot[{f[x], x^2}, {x, -Pi, Pi}, PlotStyle → {Dashing[{}], Dashing[{0.01, 0.01}]}]
```



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