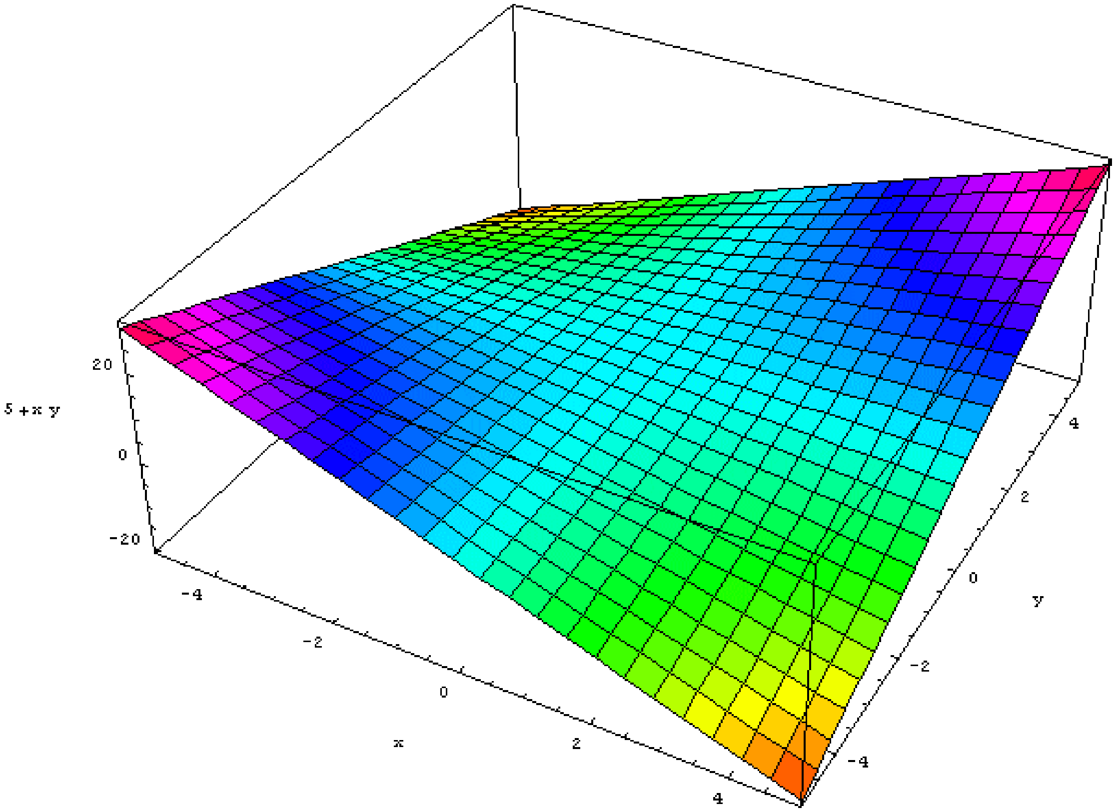
Mathematica (12)

Mathematica can be used to calculate partial derivatives. The built in function D[f[x,y], x] calculates the partial derivative of f(x, y) with respect to x. (Note that D[f[x], x] also calculates the ordinary derivative of the function f(x).)

Consider the problem of minimizing the distance from the origin to the plane z = xy + 5. First we generate a 3 dimensional plot of z.

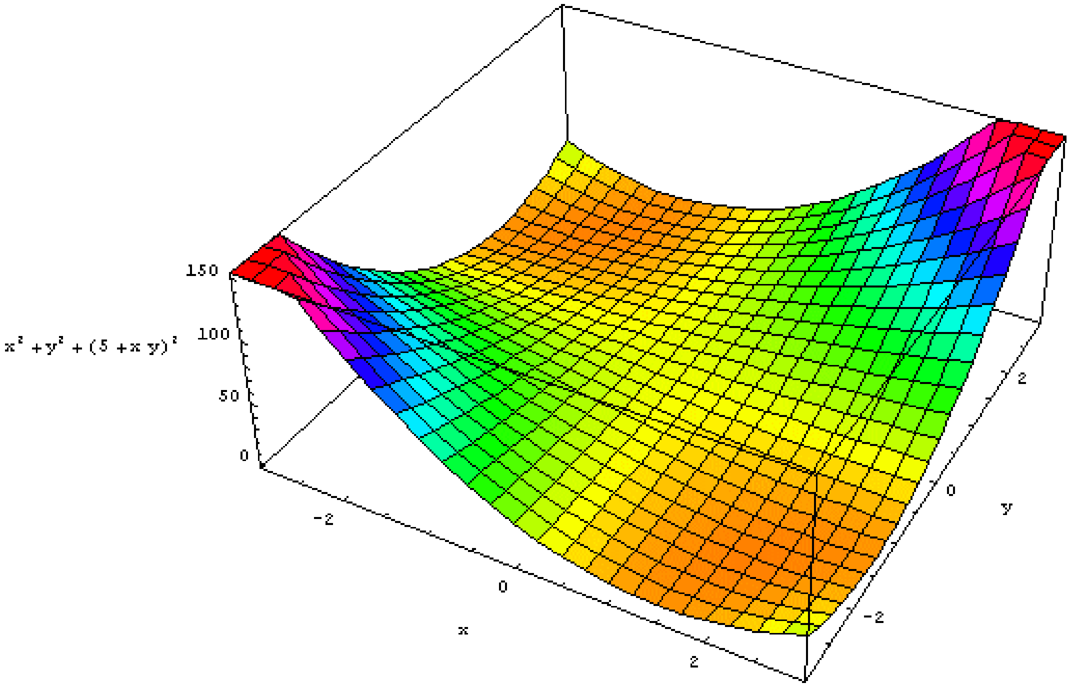
```
In[15]:= z = x y + 5;
         Plot3D[z, {x, -5, 5}, {y, -5, 5}, ColorFunction → Hue, AxesLabel → {x, y, z}]
```



The distance d is the square root of $x^2 + y^2 + z^2$. Since the distance squared will be minimized when the distance is minimized, we choose to minimize the distance squared because it is easier to work with.

$d^2 = x^2 + y^2 + (xy + 5)^2$ is the distance squared function as a function of x and y. We plot this as well. Note that z has already been defined so Mathematica will make the appropriate substitution.

```
In[19]:= dd = x^2 + y^2 + z^2;
         Plot3D[dd, {x, -3, 3}, {y, -3, 3}, ColorFunction → Hue, AxesLabel → {x, y, dd}]
```



It is hard to tell from this plot where the minima are, but if one follows the contour lines and colors it looks like there might be a minimum at about 2, -2 and –2, 2.

We now find the partial derivatives of dd(x,y) with respect to x and y and set them equal to 0. Then we solve them for x and y.

We then convert these solutions into implicit equations for x and y equal to 0. Since they are both equal to 0, there are equal to each other. We then solve for x and y when these equations are equal to each other. We then substitute these solutions back into one equation and pick the real solutions. These need to be tested to see which correspond to the minimum distances.

The details are given below.

```
D[dd, x]
```

```
2 x + 2 y (5 + x y)
```

```
D[dd, y]
```

```
2 y + 2 x (5 + x y)
```

```
Solve[2 x + 2 y (5 + x y) == 0]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

$$\left\{\left\{x \to -\frac{5\,y}{1+y^2}\right\}\right\}$$

```
Solve[2 y + 2 x (5 + x y) == 0]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

$$\left\{\left\{y \to -\frac{5\,x}{1+x^2}\right\}\right\}$$

```
Solve[ x + x y^2 + 5 y == y + y x^2 + 5 x]
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. More…
```

$$\left\{\left\{x \to -\frac{4}{y}\right\}, \{x \to y\}\right\}$$

```
Solve[x + x^3 + 5 x == 0]
```

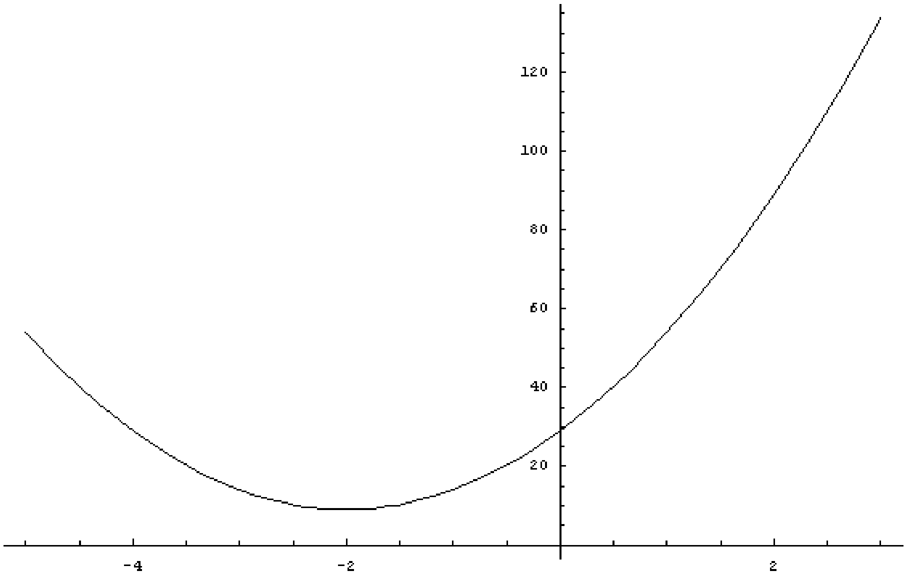
$$\left\{\{x \to 0\}, \left\{x \to -i\sqrt{6}\right\}, \left\{x \to i\sqrt{6}\right\}\right\}$$

```
Solve[ -4 / y - 4 y + 5 y == 0]
```

$$\{\{y \to -2\}, \{y \to 2\}\}$$

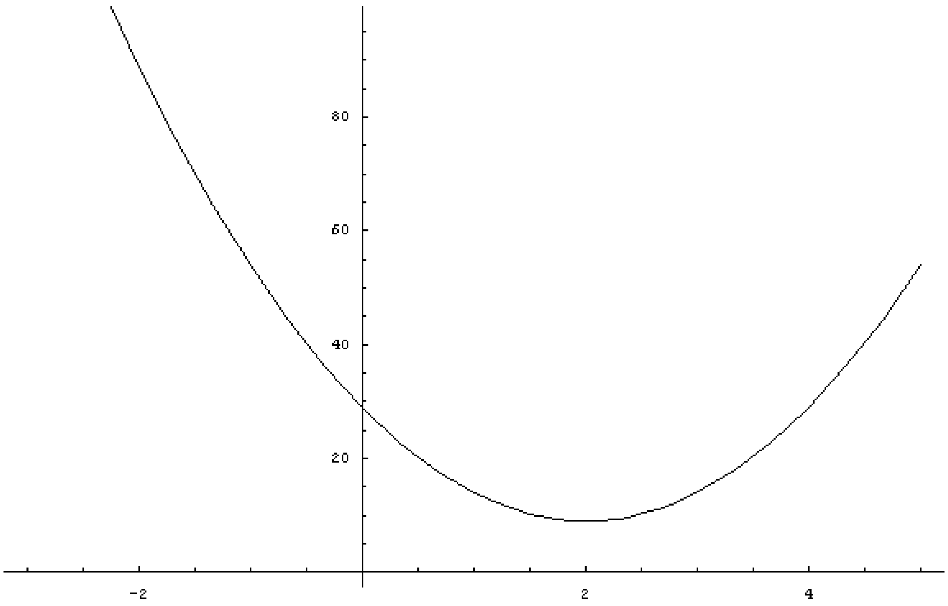
Now we plot dd for y = 2 as x varies, and we find a minimum at x = -2 as expected from x = -4/y.

```
y = 2;
Plot[dd, {x, -5, 3}]
```



We also plot dd for x = -2 as y varies, and we confirm that there is a minimum at y = 2.

```
Clear y;
x = -2;
Plot[dd, {y, -3, 5}]
```



The solution x = y = 0 gives z = 5 and a distance squared of 25 or a distance of 5. The solution y = 2, x = -2, z = 1 or y = -2, x = 2, z = 1 gives a squared distance of 9 or a distance of 9. Thus these are candidates for being the minima. To check, we make a table of values of y between 1.8 and 2.2 and x between –2.2 and –1.8. It is clear from these lists of values that 9 is, in fact, the minimum squared distance.

```
Clear x;
Clear y;
Table[dd, {x, -2.2, -1.8, 0.1}, {y, 1.8, 2.2, 0.1}]

{{9.1616, 9.1224, 9.2, 9.3944, 9.7056},
 {9.1384, 9.0401, 9.05, 9.1681, 9.3944}, {9.2, 9.05, 9., 9.05, 9.2},
 {9.3464, 9.1521, 9.05, 9.0401, 9.1224}, {9.5776, 9.3464, 9.2, 9.1384, 9.1616}}
```