

Mathematica (2)

Mathematica can be used to find symbolic or numerical values of derivatives of functions. To evaluate the derivative of $f(x) = a x^n$ we can do the following:

```
D[a x^n, x]  
a n x-1+n  
f1[x_] = %  
a n x-1+n  
f1'[x]  
a (-1 + n) n x-2+n  
D[a x^n, {x, 3}]  
a (-2 + n) (-1 + n) n x-3+n
```

The first (bold) line shows that the command for differentiation is the built in function `D[]` which has two arguments. The first of these is the function to be differentiated and the second is the variable that the function is to be differentiated with respect to.

The second (bold) line defines the function `f1` to be the result of the previous operation. (The symbol `%` means “put the result of the previous operation here”.)

The third (bold) line uses the `'` (apostrophe) operator to differentiate the function `f1[x]`.

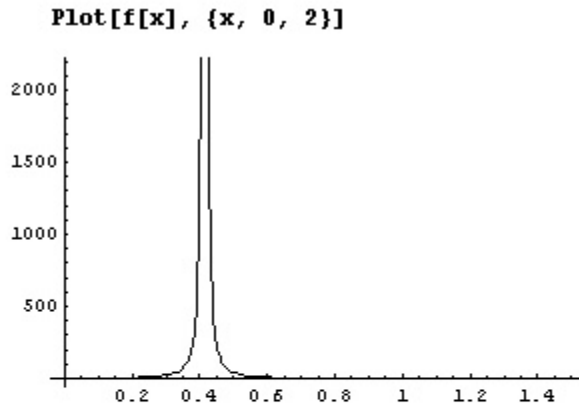
The fourth (bold) line uses the `D[]` operator again to find the third derivative. This time the second argument is a list consisting of the variable and the order of the derivative.

Mathematica can do much more complicated derivatives. Consider the following:

```
D[(x^2 - 1) / (x^2 + 2 x - 1), x]  
-  $\frac{(2 + 2x)(-1 + x^2)}{(-1 + 2x + x^2)^2}$  +  $\frac{2x}{-1 + 2x + x^2}$   
Simplify[%]  
 $\frac{2(1 + x^2)}{(-1 + 2x + x^2)^2}$   
f[x_] = %  
 $\frac{2(1 + x^2)}{(-1 + 2x + x^2)^2}$ 
```

Notice that the second bold line uses the Mathematica built in function `Simplify[]` to simplify the results of the previous expression (%). The third bold line defines the derivative function `f[x]`.

Now we can look at some of the properties of the derivative. First we plot it and note that it blows up at about 0.4:



- Graphics -

```
N[f[2]]
```

```
0.204082
```

```
Solve[x^2 + 2 x - 1 == 0]
```

```
{{x -> -1 -  $\sqrt{2}$ }, {x -> -1 +  $\sqrt{2}$ }}
```

Then, we evaluate it at $x=2$. Finally, we look for roots of the denominator by using the Mathematica built in function `Solve[]`. Note that the equation uses the double equals sign `==` since the single `=` sign is used to name objects (`f[x_] = %`, `f[x_] = a x^n`) and to assign values (`x = 1`). From the second root, we see that the denominator in the expression for the derivative goes to zero when $x = 0.4142\dots$

We can also use Mathematica to solve the recursive relation we derived in class to find the square root of 2. To do this we proceed as follows:

```
Clear[x]
x[0] = 1;
x[n_] := 1/x[n-1] + x[n-1]/2;
Table[{n, x[n]}, {n, 0, 4}]

{{0, 1}, {1,  $\frac{3}{2}$ }, {2,  $\frac{17}{12}$ }, {3,  $\frac{577}{408}$ }, {4,  $\frac{665857}{470832}$ }}
```

```
N[Sqrt[2], 12]

1.41421356237

N[(17/12), 12]

1.41666666667

N[(577/408), 12]

1.41421568627

N[(665857/470832), 12]

1.41421356237
```

The first statement is new. The Mathematica built in function `Clear[]` removes and previous definitions of `x`. (You can see if the symbol `x` has any pre-existing definitions by using `?x`.) This is not necessary here, but you will probably need to use it if you have a large notebook using lots of variables.

The next two lines define the recursive function `x[n]`. (Note the use of `n_` in the function definition.) The second function definition uses the symbol `:=` which causes Mathematica to “remember” all the previous function definitions for `x[n]` so that they don’t need to be recalculated each time `x[n]` is calculated for a larger value of `n`.

The built in Mathematical function `Table[]` makes a table of two element lists consisting of the value of `n` and the value of `x[n]` specified in the first argument over the range of `n` specified in the second.

The following lines compare the value of `Sqrt[2]` to 12 decimal places found using the built in Mathematica “numeric” function `N[]`. The subsequent lines show how well the various approximations agree with the 12 decimal place square root of 2. Note that the accuracy of the approximation is increasing by approximately doubling the number of decimal place accuracy with each iteration. (If the second argument of `N[]` is left off, it defaults to 5 decimal place accuracy.)

Exercise: Find a recursive relation for the cube root of 3 and use Mathematica to evaluate it for values of n from 0 to 4.