

Mathematica (6)

Mathematica knows about complex numbers and routinely uses complex numbers when they arise in solutions of algebraic equations or in other calculations. For example when one calculates the numeric value of `ArcSin[3]`, one gets:

```
In[13]:= N[ArcSin[3]]  
Out[13]= 1.5708 - 1.76275 i
```

Mathematica also has some capability for manipulating complex numbers. These numbers have the form $a + b I$ where I represents the square root of -1 . For example:

```
In[1]:= z1 = 2 + 3 I;  
       z2 = 2 - 1 I;  
       z1 + z2  
Out[3]= 4 + 2 i  
  
In[4]:= z1 / z2  
Out[4]=  $\frac{1}{5} + \frac{8 i}{5}$ 
```

Mathematica can also work with complex numbers in polar form. For example:

```
Abs[z1]  
Out[5]=  $\sqrt{13}$   
  
Arg[z1]  
Out[6]=  $\text{ArcTan}\left[\frac{3}{2}\right]$ 
```

Here $z1$ is defined above as $2 + 3 I$ and `Abs[z1]` is the magnitude of $z1$ which is the square root of $z1$ times its complex conjugate $2 - 3 I$. The argument of $z1$ is the `ArcTan` of the imaginary part of $z1$ divided by the real part. Thus, $z1 = \text{Sqrt}[13] e^{i \arctan(3/2)}$

Mathematica can also find complex conjugates of complex numbers. Here it finds the complex conjugate of e^{ix} using the built in function `Conjugate[]`.

```
In[3]:= z = Exp[3 I];  
       Conjugate[z]  
Out[4]=  $e^{-3 i}$ 
```

Mathematica can also manipulate functions of complex numbers. For example, to see that $e^{ix} = \cos(x) + i \sin(x)$ we can use the built in function `ExpToTrig[]` as follows:

```
In[7]:= ExpToTrig[Exp[x I]]
```

```
Out[7]= Cos[x] + i Sin[x]
```

```
In[8]:= ComplexExpand[Sin[x + y I]]
```

```
Out[8]= Cosh[y] Sin[x] + i Cos[x] Sinh[y]
```

```
In[9]:= TrigToExp[%]
```

```
Out[9]=  $-\frac{1}{2} i e^{i x-y} + \frac{1}{2} i e^{-i x+y}$ 
```

```
In[10]:= ComplexExpand[Log[3 + 2 I]]
```

```
Out[10]=  $i \text{ArcTan}\left[\frac{2}{3}\right] + \frac{\text{Log}[13]}{2}$ 
```

Other examples show above involve expanding $\sin(x + iy)$, converting this result to complex exponentials, and expanding $\ln(3 + 2i)$. Note that the primary built in functions used in these manipulations are the built in functions `ComplexExpand[]`, `ExpToTrig[]`, and `TrigToExp[]`.