Mathematica (7)

Mathematica allows the user to specify types of variables. For example, one can define x and y to be real numbers using the Element[] built in function as illustrated below. Then variable z is explicitly a complex number. Then the ComplexExpand[] built in function is used to find the real and imaginary parts of functions of the complex variable z.

```
Element[{x, y}, Reals]
```

$(x \mid y) \in \text{Reals}$

```
z = x + y I;
ComplexExpand[Exp[z] Sin[z]]
```

$$e^x \text{Cos}[y]\, \text{Cosh}[y]\, \text{Sin}[x] - e^x \text{Cos}[x]\, \text{Sin}[y]\, \text{Sinh}[y] + i\left(e^x \text{Cosh}[y]\, \text{Sin}[x]\, \text{Sin}[y] + e^x \text{Cos}[x]\, \text{Cos}[y]\, \text{Sinh}[y]\right)$$

```
ComplexExpand[ArcSin[z]]
```

$$\text{Arg}\left[\sqrt{1-(x+i\,y)^2} + i\,(x+i\,y)\right] - i\,\text{Log}\left[\sqrt{\left(\left(-y+\left(4x^2y^2+(1-x^2+y^2)^2\right)^{1/4}\text{Cos}\left[\frac{1}{2}\text{Arg}[1-(x+i\,y)^2]\right]\right)^2 + \left(x+\left(4x^2y^2+(1-x^2+y^2)^2\right)^{1/4}\text{Sin}\left[\frac{1}{2}\text{Arg}[1-(x+i\,y)^2]\right]\right)^2\right)}\right]$$

```
ComplexExpand[ArcSin[x]]
```

$$\text{Arg}\left[i\,x+\sqrt{1-x^2}\right] - i\,\text{Log}\left[\sqrt{\left(\sqrt{(1-x^2)^2}\ \text{Cos}\left[\frac{1}{2}\text{Arg}[1-x^2]\right]\right]^2 + \left(x+\left((1-x^2)^2\right)^{1/4}\text{Sin}\left[\frac{1}{2}\text{Arg}[1-x^2]\right]\right)^2}\right]$$

In the last example shown above, the real and imaginary parts of the ArcSin[] function are given when the argument is a real number. Recall the ArcSin[3] example given in the last tutorial.

```
In[13]:= N[ArcSin[3]]

Out[13]= 1.5708 - 1.76275 i
```

This time, we will calculate the real and imaginary parts separately when x = 3 as shown below using the formulas obtained above from the ComplexExpand[] operation:

```
Arg[I 3 + Sqrt[1 - 3^2]]
```

$\frac{\pi}{2}$

```
Log[Sqrt[(Sqrt[(1 - 3^2)^2] Cos[1/2 Arg[1 - 3^2]]^2
    + (3 + ((1 - 3^2)^2)^(1/4) Sin[1/2 Arg[1 - 3^2]])^2)]]
```

$\text{Log}\left[3 + 2\sqrt{2}\right]$

```
ComplexExpand[ArcSin[3]]
```

$\frac{\pi}{2} - i\,\text{Log}\left[3 + 2\sqrt{2}\right]$

Note that this time, instead of getting decimal numbers we get numeric expressions that can easily be shown to have the same numeric values.

Mathematica also has the built in functions Re[] and Im[] which can be used to find the real and imaginary parts of a complex *number*. (They don't work on complex functions; use ComplexExpand[] on functions to find the real and imaginary parts.) Note that ComplexExpand[] can also be used to find the real and imaginary parts of a complex number.

```
Re[(3 + 2 I) Sin[3 I]]
```

```
-2 Sinh[3]
```

```
Im[(3 + 2 I) Sin[3 I]]
```

```
3 Sinh[3]
```

```
ComplexExpand[(3 + 2 I) Sin[3 I]]
```

$(-2 + 3\,i)\,\text{Sinh}[3]$