(Some perhaps not so)
Frequently **A**sked **Q**uestions

1. Functions: How are functions used in the "real world?"

Many of the "laws of nature" are written as differential equations, equations involving derivatives, and derivatives are functions (derived from other functions). Such equations become the foundation for theories (models) – in chemistry, the Michaelis-Menten equations that describe reaction rates; electrical circuits that have resistors and capacitors; in finance, option pricing methods; in psychology, equations that describe rates of learning.

2. Absolute Value: How do you solve for the domain of a function with absolute value, such as the function in problem 35, page 9?

A way that you can always use to solve equations and inequalities involving absolute value is to use the definition of absolute value (page 6) directly, which means that there are 2 cases for the expression within each different absolute value. If there is only one absolute value involved there are just the two cases.

As an example, say the inequality or equation involves $|2x+3|$. The 2 cases are $2x+3>=0$, in which case $|2x+3|=2x+3$, and $2x+3<0$, in which case $|2x+3|=-2x-3$.

If there are _two_ absolute value involved, there are _4_ cases. As an example, say the inequality or equation involves $|2x+3|$ and $|1-x|$. Then the 4 cases are:

$2x+3>=0$ and $1-x>=0$;
$2x+3>=0$ and $1-x<0$;
$2x+3<0$ and $1-x>=0$;
$2x+3<0$ and $1-x<0$.
(Show that these 4 cases reduce to 3, with one case being impossible.)

There are simpler approaches, tailored to different contexts, but you would be assured of reducing the problem to one that does not involve absolute values by using the above approach.

3. Inverse Functions: I noticed these:we could have a function $f(x)=x^n$, and $g(x)=x^{(-n)}$. This means that g is the inverse function of f. Can we therefore conclude that the domain of the function f is the range for the function g?

You are basing your thinking on an error in thinking about inverse functions: The two functions you use are _not_ inverses of each other. While it is true that f(x) is the reciprocal (inverse) of g(x) for any number x, this is not the meaning of functions being inverses. One function f is the inverse of another function g if each "undoes" what the other "does" to any number, which in symbolic form is: f(g(x))=x and g(f(x))=x. Try using this on f and g; you'll see that f(g(x)) is not the same as x.

4. Definition of Function: The formal definition of a function is that for every element in a set of x there is only one y and vice versa but there are functions that do have two y values for one x such as absolute value function.

In answer to your question, you've read the definition wrong - there is no "and vice versa" in the definition of function. Once you add the "and vice versa" you get a special kind of function, called a *one-to-one* function. See page 148 for more information.

5. Nervousness: You mentioned derivatives several times in class today, and so I'm a little nervous now. As such, I was wondering that if derivatives are a pre-cal topic, is there any chance you might be willing to point me in the direction of a few resources which might refresh my memory?

Don't be nervous - derivatives are a calculus topic, the major calculus I topic, not a pre-calculus topic. What I was trying to say today is that I am jumping into a complex idea (because i think it's exciting) and it's natural that the idea will come slowly to you - but it will come to you - if you keep working at it and asking questions. For pre-calculus refresher problems, see the link under RESOURCES in the left column on the course home page.

6. Concerning Example 10 on page 103, why does

C'(500)=5+0.02(500)=$15/item

give us the rate at which costs are increasing with respect to the production level when x=500?

I know it is a derivative and that means it is a rate, but is the rate at which the costs are increasing $15/item? It seems that even though we are taking the derivatve we are still finding the cost of one item when the production level is at 500 items, so why is that equal to the rate at which the costs are increasing?

The confusion stems, at least in part, from the derivative being the "instantaneous" rate of change. If we go back to the meaning of the derivative, $C'(500) = \lim_{h \to 0}[\frac{C(500+h)-C(500)}{h}]$. This implies that for "small" values of h, we have an approximation $C'(500) \approx [\frac{C(500+h)-C(500)}{h}]$. If we solve the approximation just like we solve an equation, we get $C(500+h)-C(500) \approx C'(500)(h)$, and then $C(500+h) \approx C(500)+C'(500)(h)$. Finally, if we consider h=1 to be small, plug that into the approximation, and replace C'(500) by 15, we get $C(501) \approx C(500)+15$. I.e., the cost of producing the 501st item is approximately $15. However, the actual cost of producing the 501st item is $C(501)-C(500)=15.01$, in dollars. In the calculus framework, 15.01 is the average rate of change in cost over the interval [500,501] and is the actual cost of producing the 501st item, while 15 is the instantaneous rate of change at x=500.

More to come on the mathematical modeling process that this example illustrates!

7. The definition of limit says that for each positive number epsilon, there is a positive number delta so that abs(f(x)-L) <epsilon happens if 0<abs(x-a) < delta. I think that putting these inequalities into words could help me understand this. The book states how "f(x) can be made to lie within any preassigned distance from L by requiring that x be within a specified distance delta". I understand this, but I do not see how it relates to those inequalities shown above.

In answer to your question, I'll try to make the correspondences between words and symbols more explicit. To add a third perspective, look at the diagrams at the top of page 32. I believe that all the notation is consistent.

abs(f(x)-L) <epsilon corresponds to:
f(x) can be made to lie within any preassigned distance from L

The preassigned distance is represented by epsilon - On page 32, you see that epsilon is the radius of the interval about the limit L

0< abs(x-a) < delta corresponds to:
that x be within a specified distance delta

The specified distance is represented by delta - On page 32, you see that delta is the radius of the interval about the fixed number a.

8. My question is: Can we **change** the precise definition into this way: The limit (as x approaches a) of $f(x)=L$ if for every number $q>0$ there is a corresponding number $z>0$ such that if $|f(x)-L|<q$ then $0<|x-a|<z$.
If not, what's the difference and why there is a difference between them?

In answer to your question: No, it isn't equivalent to the definition given in class/in the book. I could say that the reason that your definition is different is because a statement is not equivalent to its converse, but that is probably not too illuminating. So, I'll give you the following problem to solve, which will answer your question in a way that I hope is both fun and illuminating!

Here is your problem: Let f(x)=x2. Using your definition, not the book's, do each of the following:

a. Prove that the limit, as x approaches 0, of f(x) is 0, as it should be.

b. Prove that it is NOT the case that the limit, as x approaches 2, of f(x) is the number 4, which is definitely not as it should be!

c. Prove that the limit, as x approaches 0, of f(x) is -5, which is definitely not as it should be! (Notice that I'm claiming that, by your definition, f has two different limits [actually infinitely many different limits] as x approaches 0.)

9. How do you know if a function is a combination of three funcitons? How do you know how many functions you need to split the main function into in order to use the chain rule?

There is no procedure for determining the decomposition, but ... start with x and look at the expression for the function and determine what first is done to x – for example, if the expression is sin(e^x), the first function applied is the exponential and the second is the sine, but if the expression is e^(sin(x)), the first function applied is the sine and the second is the exponential.

The way you'll know whether it's two, three, more, or something else, will come from how many steps get you to the final expression. For the examples above, there are only two steps, but for sin(e^[2x]), sin(2e^[x]), and 2sin(e^[x]), there are three.

10. How do you or can you use the chain rule with a complex quotient function that can not be simplified?

If the division is the last operation performed in the formation of the function, you need to apply the quotient rule first and then deal with derivatives of any composite parts.

11. When performing the chain rule with multiple "chains" do you work on the first two first, and then use the new value to chain with the third one and so forth, or do you work with them all at the same time?

You can do them all simultaneously, as I illustrated in class, or you can do just two at a time, and then repeat the two-at-a-time again as many times as needed. As long as it is only the chain rule that is needed, it seems easier to me to put all the links together at once.

12. I would like to clarify something I'm not quite sure about. Say the inside function has an inside function within it (so that there are three functions within one another). When you are taking the derivative of the inside functions, which do you take the derivative of first? The inside inside one, or the inside one?

The order of the derivatives doesn't matter, but the order of the composition matters because you need to have the function to be integrated expressed in the form: f(g(h(x))) or
x->h(x)=u->g(u)=v->f(v)=y.
The factors of the product (dy/dv)(dv/du)(du/dx) may be written in any order, but each of those three factors must appear. Also, it is critical for each of the three functions what variable it is differentiated with respect to. (u wrt x, v wrt u, and y wrt v)

13. What type of function(s) is the chain rule best suited for?

The only thing that you can apply the chain rule to is a composite (chain) function; for any function, several rules may apply; there may be a part that is a sum or product or quotient, and each part requires its own rule.

14. Is there a limit to how many links can be made using the chain rule or does it all depend on the number of terms in the function?

There can be any number of links, well any finite number. (Hmm; could there be an infinite number of links in a function!?) So, yes, you can just string the composite parts out in a chain with any number of links and the rule applies.

15. In the chain rule, what happens to a constant? Does it become it's own function like u=2 within F=f`((x)u) or do we take it out first because when it becomes a derivative it will be 0?

A constant by itself can never be a part of a composite function. It could be a part, as in .... u->2u ... but that is more easily dealt with by using the constant multiple rule, as noted in the first example function composed by the class.

16. Today, you used y=sin2x as a function. You used the chain rule for differentiation but can I differentiate using other methods such as the product rule or U substition? Sometimes, I get confused because I do not know when I can only use the chain rule and when I can't.

You can only use the product rule when you have a product and sin2x is not a product. The expression u substitution generally applies to integration, not differentiation. I think what you are missing is being able to "read" an expression for a function and know what operations and functions are implicit in such expressions. I'll try some illustrations below:

Consider the following three strings of symbols and what they mean:

sin(2x)   (sin2)(x)   (sin)(2x)   sin2x

The first one, sin(2x), would be read, "the sine of 2x," which is a composite function: x->2x=u->sin(u)=y or f(g(x)), with g(x)=2x and f(u)=sin(u). Its derivative is f'(g(x))*g'(x)=cos(2x)*2 by the chain rule.

The second one, (sin2)(x), would be read, "the sine of 2 multiplied by x," which is a constant multiple, the number sin(2), of the linear function y=x: Its derivative is sin(2)*1 by the constant multiple rule.

The third one just doesn't make any mathematical sense; the string of characters sin doesn't mean anything by itself, although you may notice me talking about the sine function, just as I talk about exponential functions and linear functions, and polynomial functions. The sine function needs to be applied to something.

The fourth one is what you wrote. Using the convention that functions like the sine have priority over multiplication, sin2x would mean sin(2)*x, the same as the second expression. But, I suspect that you intended it to mean sin(2x), the same as the first expression.

The moral of the story is that you use the chain rule if and only if you have a composite function to differentiate - but it isn't always easy to determine whether you actually have a composite function by looking at the expression for the function; it may take careful analysis and the development of good intuition.

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