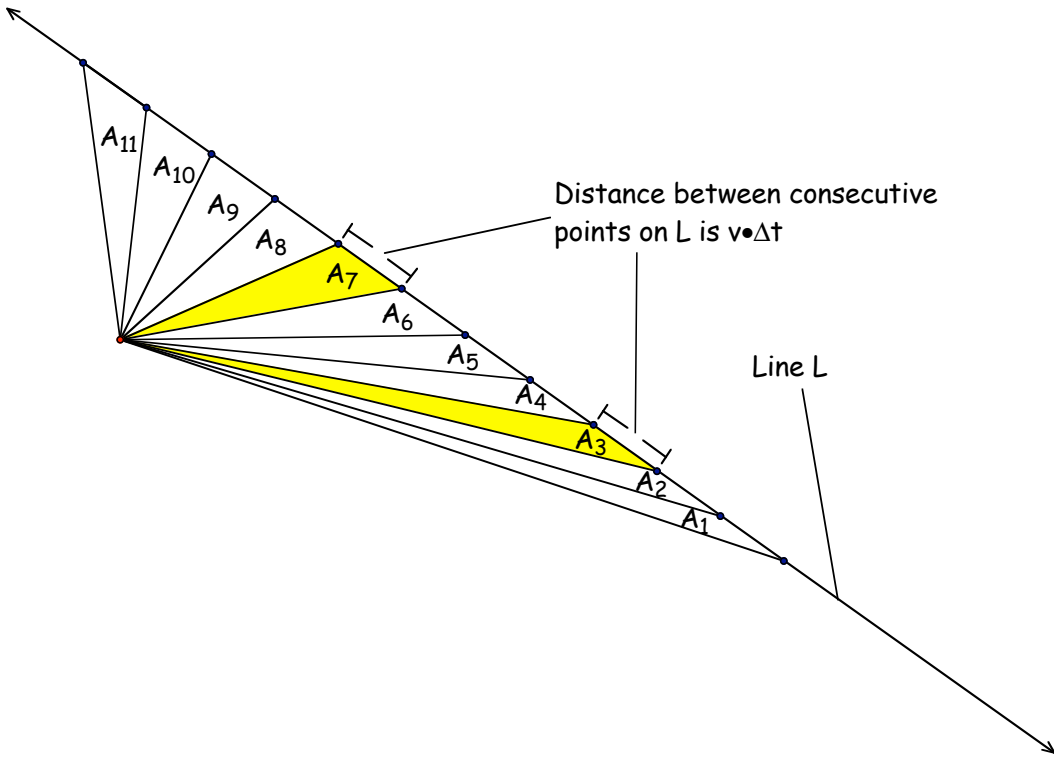
Problems
for
Mathematics Majors/Minors

1. Graph the inequality $|x|+|y|\leq 1$.
2. Graph the inequality $|x+y|\leq 1$.

3. The picture below illustrates the motion of a small object moving with a constant velocity v along a straight line L past a stationary point. The dots on the straight line show positions that are Δt units of time apart, which implies that the dots are $v\Delta t$ units of distance apart. Show that all the areas A_1, A_2, A_3, … are equal. (A_3 and A_7, are shaded just for illustration purposes.)



4. This problem involves what are called the two *hyperbolic trig* functions, cosh and sinh (pronounced to rhyme with *pinch*). For any real number x,

$\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$, where e is the number that is the base of the natural logarithm and is approximately 2.718.

a. Show that, just as (cos(x),sin(x)) always lies on the unit circle, $x^2 + y^2 = 1$, (cosh(x),sinh(x)) always lies on the unit hyperbola, $x^2 - y^2 = 1$, and hence the hyperbolic trigonometric functions satisfy the identity $\cosh^2(x) - \sinh^2(x) = 1$.

b. There are many trigonometric identities for the circular trig functions of which you are probably aware. (Many of these identities are listed just inside the front cover, on a clip-out page numbered $\boxed{2}$.)

   Investigate the possibility of one or more of these trigonometric identities having a hyperbolic trig analogue, a hyperbolic trig identity that is analogous to the trig identity. (Part a) above shows a hyperbolic trig identity analogous to the fundamental trig identity $\cos^2(x) + \sin^2(x) = 1$.

5. On the automatic doors from Clapp to Kendade (2nd and 3rd floors – maybe the 1st, too), when you go from Clapp to Kendade, you see a warning: $\frac{\partial(Ramp)}{\partial x} \neq K$. Explain what this means in terms of calculus and the ramp that connects the two buildings.

6. A professor is now 3 times as old as her student was when the professor was as old as the student is now. If the sum of their ages is now 55, how old is each now?