Math 101 X(02) Practice Exam
in preparation for 2nd midterm, April 14
closed book

1.
(a) Find functions f and g such that $F(x) = \sin(x^3 - x) = f(g(x))$.
(b) Find the derivative $F'(x)$, and explain how representing F as the composition of f and g helps you do this.

2.
(a) Explain in a short paragraph why

$$\frac{d}{dt}(R^2) = 2R\frac{dR}{dt} \tag{1}$$

You may use diagrams and formulae as appropriate.
(b) How fast is a circular area growing when it is 2 m in radius if the radius is growing at 0.1 m/s?

3. Find derivatives of the following functions:
(a) $\ln(\sqrt{x^2+1})$
(b) $e^{sin(x)}$
(c) $cos(e^{-x})$

4. A 10-foot ladder leans against a wall. How should it be positioned so that the triangular region bounded by the ladder, the floor, and the wall, has maximum area?
(a) Give a common sense argument for a plausible answer.
(b) Give a careful solution to the problem using calculus, and making your reasoning clear.

5. Two dogs are running back and forth, one on A Street and one on First Avenue, which we may as well consider the x and y axes respectively. The two streets intersect at the origin of coordinates. In these terms the first dog's position as a function of time is $x = \sin(t)$, and the second dog's position is $y = \cos(t)$. At what rate is the distance changing between the two dogs? Make your reasoning clear.

6. If a function $y = f(x)$ is such that $f'(x) = x$, then we know the slope of the tangent line to the graph at every point (x, y) of the plane.
(a) Make a sketch of the plane with these slopes indicated by little lines.
(b) Sketch the graph of a typical solution $y = f(x)$.
(c) Give a formula for a typical solution $y = f(x)$.