Practice Exam Solution

1.
(a) $F(x) = f(g(x))$ with $g(x) = x^3 - x$, $f(u) = \sin(u)$.
(b) $F'(x) = (3x^2-1)\cos(x^3-x)$. Since $F$ is the composition of $f$ and $g$, the chain rule says $F'$ is the product of the derivatives of $f$ and $g$: $F'(x) = f'(u)g'(x)$, where $u = g(x)$.

2.
(a) The $t$ derivative indicates that $R$ is being regarded as a function of $t$. Thus $R^2$ is a function of $t$ by composition: given $t$, you find $R(t)$, and then you square the result. The composition is $R^2 = f(R(t))$, where $f(u) = u^2$. By the chain rule, the derivative is $f'(u)R'(t)$, with $u = R(t)$, i.e. $2R(t)R'(t)$.
(b) $\frac{d}{dt}\pi R^2 = \pi \cdot 2RR' = 2\pi \cdot 2 \cdot 0.1 = 0.4\pi\, m^2/s$.

3.
(a)$\frac{x}{x^2+1}$
(b)$\cos(x)e^{\sin(x)}$
(c)$e^{-x}\sin(e^{-x})$

4.
(a) Since the ladder makes a very narrow triangle if it is nearly lying down, or if it is nearly straight up, it should probably be at an intermediate angle, around $45^o$, to make the maximum triangular area.
(b) Regard the wall and floor as the $x$ and $y$ axes, intersecting at the origin, and denote by $x$ and $y$ the non-zero coordinates of the ends of the ladder. Then $x^2 + y^2 = 100$, and the area we are to maximize is $A = xy/2$. Looking for critical points, we find the derivative $A' = (y + xy')/2$, and differentiating the identity relating $x$ and $y$, we find $2x + 2yy' = 0$. Thus $y' = -(x/y)$, and so $A' = (y - x^2/y)/2$. This is zero when $x^2 = y^2$, i.e., $x = y$, so the guess in (a) was right: the ladder should be at $45^o$. We have $x^2 = y^2 = 50$, in this case, so $A = xy/2 = 25\, ft^2$.

5. The first dog is at the point $(\sin(t), 0)$, and the second dog is at the point $(0, \cos(t))$ at the time $t$. Thus the distance between them is

$$D(t) = \sqrt{\cos^2(t) + \sin^2(t)} = \sqrt{1} = 1 \qquad (1)$$

Since this is the constant function, its derivative is zero: the distance between them is not changing, even as they run around.

6.
(a) See link on "written homework" page.

(b) See link on "written homework" page.
(c) $y = \frac{1}{2}x^2 + C$