

Math 202(X02) Practice Exam Dec. 5, 2003

Actual final exam will be through the exam center. You may have one hand-written sheet of notes, otherwise CLOSED BOOK.

Problem 1. (a) Find the Taylor series for

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad (1)$$

(b) Find the Taylor series for

$$\cosh(x) = \frac{d \sinh(x)}{dx} \quad (2)$$

Problem 2. What rational functions have the Taylor series:

(a)
$$1 - x^2 + x^4 - x^6 + x^8 - \dots \quad (3)$$

(b)
$$1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + x^6 + \dots \quad (4)$$

(c) What is the radius of convergence for the series in (b)? Prove it!

Problem 3. Prove that the series in the binomial expansion

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n \quad (5)$$

has radius of convergence $R = 1$ if k is not a positive integer or zero. How is the situation different if k is a positive integer?

Problem 4. Compute

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^3}}{x^3}. \quad (6)$$

by finding a series representation for the numerator.

Problem 5.

(a) Find the general solution to the ODE

$$\frac{dy}{dx} = y + x \quad (7)$$

- (b) Specialize your solution in (a) to solve the initial value problem $y(0) = 0$.
(c) Give the power series representation for your solution in (b). (NOTE: You can do part (c) even if you couldn't do part (b))

Problem 6. Use

$$\int_0^x \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1}(x) \quad (8)$$

to find the first 3 nonzero terms of the power series for $\sin^{-1}(x)$.

Additional practice problems: see last homework set (problems for discussion).