Practice Exam

1. Find the sum of $\sum_{k=2}^{\infty} \left(\frac{4}{5}\right)^k$. Note the value of the index k for the first term.

2. Use the comparison test to determine whether $\sum_{n=0}^{\infty} \frac{1}{n+2^n}$ converges or diverges. The series you compare with is up to you, so be clear what series you choose, and make your reasoning clear.

3. Consider the function $f(x) = \ln(1+x)$.

(a) By evaluating f and several of its derivatives at $x = 0$, find the Taylor series for f about $x = 0$.

(b) Use the ratio test to find the radius of convergence of this series.

(c) Find an approximation to $\ln(1.5)$ by summing the first three terms of this series at the appropriate value of x.

(d) According to the alternating series test, how far off could this sum be from the exact value of $\ln(1.5)$?

4. TRUE/FALSE questions. For each statement, write whether it is true or false, and provide a short explanation if it is true, or a counterexample if it is false.

(a) If $\sum a_k$ is an infinite series, and if $\lim_{k\to\infty} a_k = 0$, then the series converges.

(b) If a series $\sum a_k$ converges, then $\sum |a_k|$ converges.

(c) If a series $\sum a_k$ diverges, then $\sum |a_k|$ diverges. (Hint: consider the contrapositive.)

5. (a) Use the binomial series to find the Maclaurin series of $f(x) = 1/\sqrt{1+x^3}$.

(b) Use part (a) to evaluate $f^{(9)}(0)$.