Math 324 Problem 12 Solution 11 March 2004

$$\frac{dp^i}{dt} = \frac{d}{dt}\int_{D_t} \rho V^i \, dx\,dy\,dz$$

$$= \int_{D_t} \frac{\partial}{\partial t}(\rho V^i)\,dx\,dy\,dz + \int_V \rho V^i \, dx\,dy\,dz$$

$$= \int_{D_t} \left[\frac{\partial}{\partial t}(\rho V^i) + \frac{\partial}{\partial x^j}(\rho V^i V^j)\right] dx\,dy\,dz$$

$$= \int_{D_t} \left\{ \underbrace{\left[\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V^j)}{\partial x^j}\right]}_{=0 \text{ by continuity equation}} V^i + \rho \frac{\partial V^i}{\partial t} + \rho V^j \frac{\partial V^i}{\partial x^j} \right\} dx\,dy\,dz$$

$$= \int_{D_t} \rho \left(\frac{\partial V^i}{\partial t} + V^j \frac{\partial V^i}{\partial x^j}\right) dx\,dy\,dz$$