Math 324 Problem 13 Solution 23 Mar 2004

$$\mathcal{L}_V (V_j dx^j)\left(\frac{\partial}{\partial x^k}\right) = V\left(V_j \delta^j{}_k\right) + V_j dx^j\left(\left[\frac{\partial}{\partial x^k}, V^i \frac{\partial}{\partial x^i}\right]\right)$$

$$= V V_k + V_j dx^j \left(\frac{\partial V^i}{\partial x^k}\frac{\partial}{\partial x^i}\right)$$

$$= V^i \frac{\partial V_k}{\partial x^i} + V_j \frac{\partial V^i}{\partial x^k}\delta^j_i$$

$$= V^i \frac{\partial V_k}{\partial x^i} + V_i \frac{\partial V^i}{\partial x^k}$$

$$= V^i \frac{\partial V_k}{\partial x^i} + \frac{1}{2}\frac{\partial}{\partial x^k}\left(|V|^2\right) \quad \text{(Cartesian)}$$

Thus $$\mathcal{L}_V (V_j dx^j) = V^i \frac{\partial V_k}{\partial x^i} dx^k + d\left(\frac{1}{2}|V|^2\right)$$

Note this combination occurs in the convective derivative