Math 324 Problem 17 Solution 5 Apr 2004

The shear stress on the wall of the pipe in Poiseuille flow is, using $v = \frac{2Q}{\pi R^4}(R^2 - r^2)\frac{\partial}{\partial z}$

$$2\eta\, e\left(-\frac{\partial}{\partial r}, \frac{\partial}{\partial z}\right)$$ (evaluating on $-\frac{\partial}{\partial r}$ and looking at the z-component)

$$= 2\eta \cdot \frac{1}{2} \mathcal{L}_v g\left(-\frac{\partial}{\partial r}, \frac{\partial}{\partial z}\right)$$

$$= \eta\, g\left(\left[-\frac{\partial}{\partial r}, v\right], \frac{\partial}{\partial z}\right)$$

$$= \eta \cdot \frac{2Q}{\pi R^4}(2r)\Big|_{r=R} = \frac{4Q\eta}{\pi R^3}$$

This is a force per unit area in the z-direction

For data on the aorta ($Q \doteq 10^{-4}$, $R \doteq 10^{-2}$, $\eta \doteq 10^{-3}$)

This is $\frac{4 \cdot 10^{-4}\, 10^{-3}}{\pi \cdot 10^{-6}} \doteq 0.1 \; N/m^2$

It is about one millionth of the normal stress due to atmospheric pressure ($10^5 N/m^2$), but that, after all, is quite a large stress by biological standards.