Math 324 Problem 19 Solution 12 April 2004

Find a Stokes flow with $p = z$.

This means find $\vec{V}$ such that

(1) $\eta \nabla^2 V_x = 0$

(2) $\eta \nabla^2 V_y = 0$

(3) $\eta \nabla^2 V_z = 1$

(4) $\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$

One possible answer is the Poiseuille flow we have already found: $V_z = \frac{1}{4\eta}(x^2+y^2)$, $V_x = V_y = 0$

There are many possibilities, though; for example, taking $V_z = \frac{1}{2\eta} z^2$, solving (3), implies $\frac{\partial V_z}{\partial z} = \frac{z}{\eta}$. Then $V_x = -\frac{xz}{2\eta}$, $V_y = \frac{-yz}{2\eta}$ solves (4), and these also obey (1) and (2).