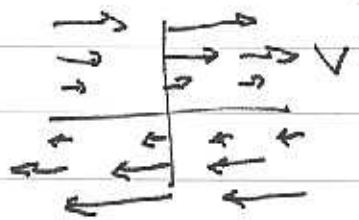
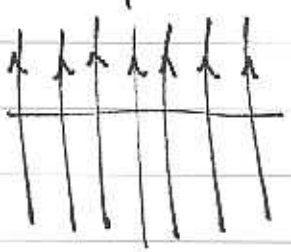
Math 324 Problem 4 Solution 11 Feb 2004

Given $U = \frac{\partial}{\partial y}$ $V = y\frac{\partial}{\partial x}$, how does V change U?

$$\mathcal{L}_V U = [U, V] = \frac{\partial}{\partial y}\left(y\frac{\partial}{\partial x}\right) - y\frac{\partial}{\partial x}\frac{\partial}{\partial y} = \frac{\partial}{\partial x}$$

In pictures, the integral curves of U are vertical, and the shearing flow of V makes them diagonal:





flow of U becomes thick under the action of the shearing flow V.

Thus U, which was $\frac{\partial}{\partial y}$, becomes $\frac{\partial}{\partial y} + \epsilon\frac{\partial}{\partial x}$,

just what the Lie derivative computation above said would happen. The change is in the direction $\frac{\partial}{\partial x}$, and it is the same everywhere. That is, the new flow lines are still straight, but no longer vertical.