

Maths 324

Problem 6 Solution

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$$v_j dg^j \leftrightarrow g^{ij} v_j \frac{d}{dg^i} \quad (\text{canonical isomorphism})$$

$$w_k dg^k \leftrightarrow g^{lk} w_k \frac{d}{dg^l} \quad (\text{similarly})$$

so if this is to be an isometry, we must require

$$g(v_j dg^j, w_k dg^k) = v_j w_k g(dg^j, dg^k)$$

to be the same as

$$\begin{aligned} g(g^{ij} v_j \frac{d}{dg^i}, g^{lk} w_k \frac{d}{dg^l}) &= g^{ij} g^{lk} v_j w_k g\left(\frac{d}{dg^i}, \frac{d}{dg^l}\right) \\ &= g^{ij} g^{lk} v_j w_k g_{il} = \delta_j^l g^{lk} v_j w_k \quad (\text{using } g^{ij} g_{il} = \delta_j^l) \\ &= g^{jk} v_j w_k \quad (\text{doing the sum over } l). \end{aligned}$$

Thus  $g(dg^j, dg^k) = g^{jk}$  : the cometric is the inverse of the metric.