Math 324 Problem 7 Solution 25 Feb 2004

(a)

$$\begin{array}{lcl} x = r\sin\theta\cos\varphi & \Rightarrow & dx = \sin\theta\cos\varphi\, dr + r\cos\theta\cos\varphi\, d\theta - r\sin\theta\sin\varphi\, d\varphi \\ y = r\sin\theta\sin\varphi & \Rightarrow & dy = \sin\theta\sin\varphi\, dr + r\cos\theta\sin\varphi\, d\theta + r\sin\theta\cos\varphi\, d\varphi \\ z = r\cos\theta & \Rightarrow & dz = \cos\theta\, dr - r\sin\theta\, d\theta \end{array}$$

(b)

$$g(dx,dx) = (\sin\theta\cos\varphi)^2 g(dr,dr) + (r\cos\theta\cos\varphi)^2 g(d\theta,d\theta) + (r\sin\theta\sin\varphi)^2 g(d\varphi,d\varphi)$$

$$= \sin^2\theta\cos^2\varphi + r^2\cos^2\theta\cos^2\varphi\cdot\frac{1}{r^2} + r^2\sin^2\theta\sin^2\varphi\cdot\frac{1}{r^2\sin^2\theta}$$

$$= \cos^2\varphi + \sin^2\varphi = 1$$

Similarly $g(dy,dy) = \sin^2\varphi + \cos^2\varphi = 1$

$$g(dz,dz) = \cos^2\theta + (r\sin\theta)^2\cdot\frac{1}{r^2} = 1$$

$$g(dx,dy) = \sin^2\theta\cos\varphi\sin\varphi + \frac{1}{r^2}r^2\cos^2\theta\cos\varphi\sin\varphi - \frac{1}{r^2\sin^2\theta}r^2\sin^2\theta\sin\varphi\cos\varphi$$

$$= \cos\varphi\sin\varphi - \sin\varphi\cos\varphi = 0$$

$$g(dx,dz) = \cos\theta\sin\theta\cos\varphi - \frac{1}{r^2}r^2\sin\theta\cos\theta\cos\varphi = 0$$

$$d(dy,dz) = \cos\theta\sin\theta\sin\varphi - \frac{1}{r^2}r^2\sin\theta\cos\theta\sin\varphi = 0.$$