Physics 315 Problem Set 12 Solutions 4 May 2004

Ch. 7 #16) The unperturbed electron in an elliptical orbit about a proton obeys

$$m_e \frac{d^2\vec{r}}{dt^2} = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

which is the Kepler problem, with $GM \to \frac{e^2}{4\pi\epsilon_0 m} = \frac{ke^2}{m}$

Thus the period $T$ in an orbit with semimajor axis $a$ obeys $T^2 = \frac{4\pi^2 m}{ke^2}a^3$ ; so the angular frequency is

$$\omega = \frac{2\pi}{T} = \left(\frac{ke^2}{ma^3}\right)^{1/2}$$

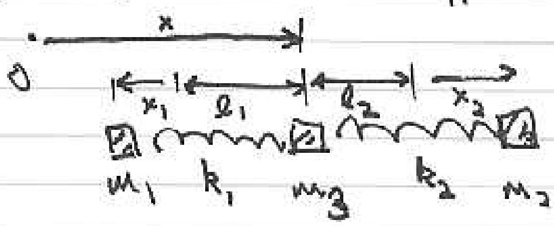
Putting in $k \doteq 10^{10}$, $e \doteq 10^{-19}$, $m \doteq 10^{-30}$, $a = 10^{-10}$, $\omega \doteq 10^{16}\,s^{-1}$

The Larmor frequency in a $B = 1\,T$ field is

$$\omega_L = \frac{eB}{2m} = \frac{1.6\times 10^{-19}}{2\times 9\times 10^{-31}} = 8.9\times 10^{10}\,s^{-1}.$$

This is the frequency with which the ellipse will precess. Since $\omega_L \ll \omega$, the Larmor approximation is valid.

Ch. 4 #40)




$\vec{r}_1 = x - x_1 - l_1$ $\quad\vec{v}_1 = \dot{x} - \dot{x}_1$ (all vectors along x-axis)

$\vec{r}_2 = x + x_2 + l_2$ $\quad\vec{v}_2 = \dot{x} + \dot{x}_2$

$\vec{r}_3 = x$ $\quad\vec{v}_3 = \dot{x}$

$$\mathcal{L} = T - V = \tfrac{1}{2}m_1(\dot{x}-\dot{x}_1)^2 + \tfrac{1}{2}m_2(\dot{x}+\dot{x}_2)^2 + \tfrac{1}{2}m_3\dot{x}^2 - \tfrac{1}{2}k_1x_1^2 - \tfrac{1}{2}k_2x_2^2$$

(equations of motion $\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{x}_i} = \frac{\partial\mathcal{L}}{\partial x_i}$ on next page)