Physics 325 Practice Exam

Actual exam will be very much like this, and will be closed book

1. The plasma membrane of a living cell is a bilayer of lipid molecules only about 4 nanometers thick. In excitable cells, like muscle and nerve cells, there is an electrostatic potential of about 100 millivolts across this membrane. The fluid on either side of the cell membrane contains mobile ions. Thus these fluid regions should be considered conductors, separated by an insulator. (a) Estimate the electric field within the membrane, assuming it is not polarizable, explaining your reasoning. (b) Also estimate the free surface charge density at the membrane surface, both in SI units and in elementary charges per square Angstrom. (c) How would your answers change if the membrane is significantly polarizable? Explain carefully.

2. Let charges q and $-q$ be located on the z-axis at $(0, 0, d/2)$ and $(0, 0, -d/2)$ respectively. (a) Find an expression for the electrostatic potential at a general point $(0, 0, z_0)$ on the z-axis. (b) Expand your expression in (a) to find the multipole expansion for the electrostatic potential for large z_0. (c) Use your expansion in (b) to give the multipole expansion for the electrostatic potential at a general point (x_0, y_0, z_0), justifying your method.

3. Let a sphere of radius R have a surface charge density

$$\sigma = \frac{3\cos^2\theta - 1}{2} k \tag{1}$$

where θ is the usual polar coordinate, and k is a constant. (Note the expression is a Legendre polynomial.) Find an expression for the electrostatic potential both outside and inside the sphere, explaining your method as you go. (Choose the potential to go to zero far from the sphere.)

4. Write the Biot-Savart Law from memory, and evaluate the contribution to the $\vec{B}$-field at a general point from a finite straight segment of current carrying wire, choosing coordinates as seems most convenient: explain your choices.

5. Use $\vec{B} = \mu_0(\vec{H} + \vec{M})$, the relationship between $\vec{H}$ and $\vec{J}_{free}$, and the definition of inductance L to explain why L for a coil is much larger if an iron core is introduced.

6. Write Maxwell's equations in a linear medium from memory, and derive Poynting's theorem starting from the rate of work done on free charge density, $\vec{E} \cdot \vec{J}_{free}$. In particular point out why the energy current density is $\vec{E} \times \vec{H}$.

7. [Some unannounced problem involving wave solutions to Maxwell's equations.]