Ch 2: 25, 28, 30, 33, 35, 39 (2 pt BONUS: 52)

Problem 2.25 Using Eqs. 2.27 and 2.30, find the potential at a distance z above the center of the charge distributions in Fig. 2.34. In each case, compute $\mathbf{E} = -\nabla V$, and compare your answers with Prob. 2.2a, Ex. 2.1, and Prob. 2.6, respectively. Suppose that we changed the right-hand charge in Fig. 2.34a to $-q$; what then is the potential at P? What field does that suggest? Compare your answer to Prob. 2.2b, and explain carefully any discrepancy.

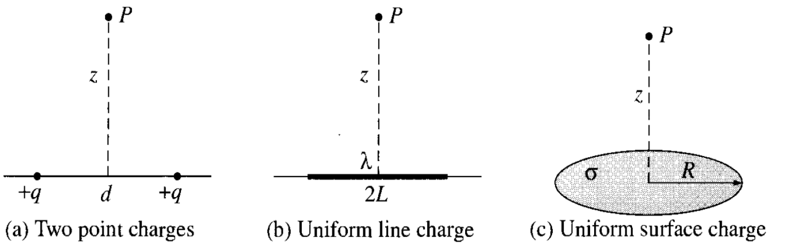

Figure 2.34

Problem 2.28 Use Eq. 2.29 to calculate the potential inside a uniformly charged solid sphere of radius R and total charge q. Compare your answer to Prob. 2.21.

Problem 2.30

(a) Check that the results of Exs. 2.4 and 2.5, and Prob. 2.11, are consistent with Eq. 2.33.

(b) Use Gauss's law to find the field inside and outside a long hollow cylindrical tube, which carries a uniform surface charge σ. Check that your result is consistent with Eq. 2.33.

(c) Check that the result of Ex. 2.7 is consistent with boundary conditions 2.34 and 2.36.

Problem 2.33 Here is a fourth way of computing the energy of a uniformly charged sphere: Assemble the sphere layer by layer, each time bringing in an infinitesimal charge dq from far away and smearing it uniformly over the surface, thereby increasing the radius. How much work dW does it take to build up the radius by an amount dr? Integrate this to find the work necessary to create the entire sphere of radius R and total charge q.

Problem 2.35 A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as in Fig. 2.48). The shell carries no net charge.

(a) Find the surface charge density σ at R, at a, and at b.

(b) Find the potential at the center, using infinity as the reference point.

(c) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as at infinity). How do your answers to (a) and (b) change?

Problem 2.39 Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b (Fig. 2.53).

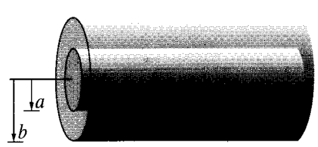

Figure 2.53

Problem 2.52 We know that the charge on a conductor goes to the surface, but just how it distributes itself there is not easy to determine. One famous example in which the surface charge density can be calculated explicitly is the ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

In this case[11]

$$\sigma = \frac{Q}{4\pi abc}\left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{-1/2}, \tag{2.57}$$

where Q is the total charge. By choosing appropriate values for a, b, and c, obtain (from Eq. 2.57): (a) the net (both sides) surface charge density $\sigma(r)$ on a circular disk of radius R; (b) the net surface charge density $\sigma(x)$ on an infinite conducting "ribbon" in the xy plane, which straddles the y axis from $x = -a$ to $x = a$ (let Λ be the total charge per unit length of ribbon); (c) the net charge per unit length $\lambda(x)$ on a conducting "needle", running from $x = -a$ to $x = a$. In each case, sketch the graph of your result.

2 pt
Bonus