

## Circular Motion and Frictional Forces

### Introduction:

In this lab you will study centripetal forces, by studying a swinging pendulum on a spring scale and a mass placed on a rotating platform. In the latter experiment you will measure the coefficient of static friction between the object and the platform.

### Centripetal Acceleration

#### Theory:

In order for an object to move in a circular path or arc, it must experience a *centripetal acceleration*, or acceleration toward the center of the circle. The acceleration  $a_c$  has a magnitude  $v^2/R$ , where  $v$  is the velocity of the object, and  $R$  is the radius of the circle. Therefore, according to Newton's second law, the net force towards the center of the circle (or *centripetal force*) is just  $\Sigma F = ma_c = mv^2/R$ . We will move an object (a pendulum bob) in a circular path and determine whether this equation holds.

#### Part I: Setup:

The assembly consists of a pendulum bob on a string, connected to a spring scale, which in turn is suspended from a pivot point on a metal rod (see figure 1).

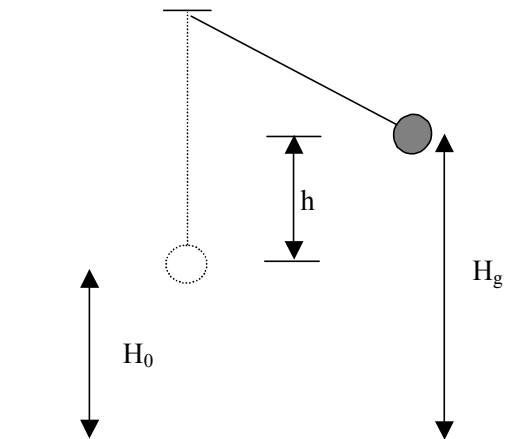
#### Procedure

With the pendulum at rest and hanging vertically, read the scale. This is the *weight* (force,  $mg$ ) of the pendulum bob. Check this by weighing your pendulum bob on a scale and multiplying by  $g$ . Next, measure the length of the pendulum, which will be the radius of the circular motion  $R$ . Finally, measure the height  $H_0$  of the pendulum from the floor.

Pull the pendulum bob to the side and release it. Notice the reading on the scale is higher as the pendulum swings past the lowest point along the swing path. Why does this occur?

Now you will measure the tension on the spring scale, at the bottom of the swing path, for various starting heights. Pull the pendulum to the side, so that it rises from the ground. Measure the height of the pendulum above the ground. Then release it, and record the tension measured by the spring scale as the pendulum bob passes the lowest point. You will do this experiment by releasing the pendulum bob from a range of heights  $h$ , relative to the position at the bottom of the swing. If  $H_0$  is the height of the bob at its lowest position, then the height relative to the ground  $H_g$  that you should release your bob from is  $H_g = h + H_0$ . Use  $0.1 R$ ,  $0.2 R$ ,  $0.3 R$ ,  $0.4 R$  and  $0.5 R$  as your release heights  $h$  relative to the bottom of the swing.

Figure 1: Lab setup



Once you have taken tension measurements from all your release heights, draw a free body diagram for your pendulum bob at the lowest point along its swing path. What is the *centripetal force*, or  $\Sigma F$ , for each trial, in terms of the tension  $T$ , and the weight  $mg$ ? Enter these into the table.

Using conservation of energy, the velocity of your pendulum bob is given by  $v^2 = 2gh$  if no energy is lost to friction during the swing. Calculate your theoretical prediction for the velocity based on  $h$  for each trial in part II. Using this velocity and the radius  $R$ , calculate the centripetal acceleration  $a_c = v^2/R$  for each trial.

Make two graphs: centripetal force  $\Sigma F$  versus velocity  $v$ , and centripetal force  $\Sigma F$  versus theoretical centripetal acceleration  $a_c$ . Have Excel generate a power-law fit on your  $\Sigma F$  vs.  $v$  graph. What is the power of the relationship? Also, fit a line to your  $\Sigma F$  vs.  $a_c$  graph. What is the slope of this line? What physical parameter does this slope correspond with? How much does the slope of this graph differ from what you expected?

**Tables:**

**Part I:**

Weight of the bob:		Height $H_0$ :	
Mass of the bob:		Radius $R$ :	

Height $h$	Experimental Measurements		Theoretical calculated values	
	Tension ( $T$ ) from scale	$\Sigma F$ (centripetal force)	$v = \sqrt{2gh}$	$a_c = v^2/R$

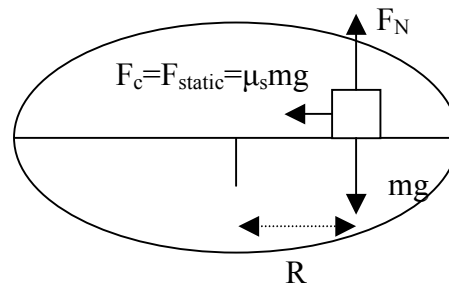
**Part II: Frictional Forces**

As you hopefully found in Part I of this experiment, an object undergoing circular motion must have a centripetal force of magnitude  $F_c = mv^2/R$  acting on it in order to maintain that circular motion. For an object sitting on a rotating platform, the force of static friction, as illustrated in Figure II, provides the required centripetal force. Static friction is the amount of force which must be overcome in order to put a body at rest into motion. In this lab you are concerned with measuring the static frictional force on an object. The maximum force of static friction is given by the equation  $F_{static} = \mu_s F_N$  where  $F_N$  is the normal force exerted by the surface and  $\mu_s$ , the

coefficient of static friction, is dependent on the type of surface contact an object makes with whatever it is resting on. When the centripetal force required by the object to stay in circular motion exceeds the maximum force of static friction, the object slides off the platform.

The linear speed  $v$  of the object depends on where the object is placed relative to the center of the rotating platform. However, all points on the platform have the same rate of rotation (i.e. they undergo the same number of revolutions per second). This rotation rate is expressed in terms of an *angular velocity*  $\omega$ , measured in units of radians/second. ( $2\pi$  radians  $\equiv$   $360^\circ$ ) The linear speed of an object located a distance  $R$  from the center of rotation is  $v = R\omega$ . The centripetal force can then be rewritten in terms of angular velocity as  $F_c = mv^2/R = mR\omega^2$ .

Figure II



### Setup:

This assembly consists of a rotating platform attached via a rubber band to a DC motor. By adjusting the voltage coming from the DC voltage source, you can adjust the speed at which the motor, and thus the rotating platform, spins. To measure the velocity of the platform, you will be using a “smart pulley” system connected to a lab computer. This pulley is essentially a rotating disk with spokes attached to a photo-detector that counts the time between the spokes. Once the platform is set in motion, you will place the smart pulley in contact with the rotating platform. By knowing the radius of the disk as well as the radius of the spinning platform, the computer software is able to accurately create a graph of the angular velocity vs. time.

### Procedure:

Your lab instructor will provide you with three different objects, and your task is to use your knowledge of centripetal force to determine the coefficients of static friction for those objects. For each, you will measure the angular velocity at which the object begins to slide off the rotating platform. From the START menu on your lab computer, select the Programs icon and then the Science Workshop icon. On the resulting display screen, find the Digital Plug icon and drag it onto the icon for the terminal which you have plugged the Smart Pulley into. From the resulting menu, scroll down and select Smart Pulley (Rotational). Next, drag the Graph icon onto the terminal icon. You will be asked which graph you want displayed; select the angular velocity versus time graph.

At this point, turn on the DC power supply and set the voltage to zero so that the motor is not turning. After weighing the object, place it on the rotating platform at a radius  $R$  which you should measure and note in the chart below. Place the smart pulley in contact with the rotating platform and select REC on the computer to start taking data. Then, slowly turn up the DC voltage until the object begins to slide. At this point, the static frictional force just became

insufficient to provide the necessary centripetal force to keep the object from moving off the rotating platform. Using the graphical readout on the computer, perform several tests on each object to figure out at which angular velocity the object begins to move. For subsequent tests, you need not place the object at the same radial distance  $R$ .

Object A	Test 1	Test 2	Test 3
Mass =			
Radius $R$			
Angular Velocity $\omega$			
$\mu_s$ (calculated)			

Object B	Test 1	Test 2	Test 3
Mass =			
Radius $R$			
Angular Velocity $\omega$			
$\mu_s$ (calculated)			

Object C	Test 1	Test 2	Test 3
Mass =			
Radius $R$			
Angular Velocity $\omega$			
$\mu_s$ (calculated)			

**Analysis:**

Using the equations for centripetal force and static friction, derive an equation for the coefficient of static friction  $\mu_s$ . Using the information you have gathered above, find the values for each object's coefficient of static friction  $\mu_s$ . Do your results for different trials on a given object agree? Find the average  $\mu_s$  for each object.

Do the values of  $\mu_s$  depend on the object's mass? Show this quantitatively then try to prove your hypothesis experimentally by adding a weight on top of Object A and determining the new coefficient of static friction. Does this experimental result agree with your theoretical prediction? If not, why?

Object A	Test 1	Test 2	Test 3
Mass =			
New Mass=			
Radius			
Angular Velocity $\omega$			
$\mu_s$ (calculated)			