PRACTICE WITH MATRICES

Tevian Dray

Identify the point (x, y) in the plane with the column vector $v = \begin{pmatrix} x \\ y \end{pmatrix}$. Then any matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ can be thought of as a linear transformation of $\mathbb{R}^2$, taking v to Av.

1. Compute $v^T v$, where v^T denotes the transpose of v.
2. Find all matrices A satisfying $(Av)^T(Av) = v^T v$.
3. Find the determinant of these matrices, that is, find $\det A$.
4. Compute vv^T.
5. Compute the trace of this matrix, that is, find $\mathrm{tr}(vv^T)$.
6. Compute the determinant of this matrix, that is, find $\det(vv^T)$.

Think about what these results mean in $\mathbb{R}^2$. Describe in words the result of question 1. What linear transformations correspond to the matrices you found in question 2. What is the significance of the determinant of A in question 3? Can you explain why the trace and determinant come out the way they do in the last two questions?