

July 8<sup>th</sup>, 2005

Question: How can you prove that some polynomial (like  $x(x-1)$  or  $x(x-1)^3$ ) “works for all  $x$ ?”

1. We can test it for a bunch of numbers (values for  $x$ ), but that won't prove it for all values of  $x$ . (L.M.)
  2. But, if we test a value of  $x$  and it doesn't “work” then we've disproved the conjecture and provided what's called a counter-example. (L.M.)
  3. Program a computer to test values of  $x$ , which wouldn't give complete understanding but might be good enough for some practical situations. (R.B.)
- Use logic and reasoning, but we may fool ourselves. (C.F.)

Reyna, Quan and Lauren on G1.

We know that when  $x = 1$  the different ways of “proper colouring” is 0. why? Because the two vertices are connected by one segment in the following case.

R \_\_\_\_\_ R WARNING : this is NOT proper coloring.

When  $x = 2$ ,  $2(2-1) = 2$ .

Does this work for any number? – We really don't know but it has worked for the numbers we've been able to test. For example:  $x=5$  and number of coloring is  $5(5-1) = 20$  ways.

How did I come up with this? Well just like set we count off continuously eliminating the previous one, here we multiply by two because for one form of coloring there is always a mirror image!

L B P O G

B P O G

P O G

O G

G

=>  $10 \times 2 = 20$  ways.

L = light green

Right now, I have only tested it as far as with 12 colors, because that is all CRAYOLA has supplied me with. And my partners visually (drawing) proved the answers my graph gave for  $x=11$  and  $x=12$ . for  $x=11$  => 11- ways, for  $x=12$  => 132 ways. *Do it out and tell me if you got the same, but remember to multiply by 2.*

C.T. and Amber G2

1.  $x(x-1)^2$  or 2.  $[(x^2-x)X] - 2x$

2 colors gave 2 variations and that doesn't work with formula 2.

R-B-R or B-R-B  $\Rightarrow$  2 variation for 2 colors.

1.  $\Rightarrow$  2 and 2.  $\Rightarrow$  0!

'x' represents the number of colors you have, (x-1) represents the colors (x) that can be with all the other colors but itself (-1) and raised to the power (# of vertices).

Rachael: TRIANGLE coloring

Conjecture  $(x^2-x)(x-2) \dots$  is wrong???? Or maybe not!

	Difference	Multiply by
When $x=1 \Rightarrow 0$		
$X=2 \Rightarrow 0$		
$X=3 \Rightarrow 6$	6	1
$X=4 \Rightarrow 24$	18	3
$X=5 \Rightarrow$	36	6
$X=6 \Rightarrow$	60	10
$X \Rightarrow$	90	15

Equation is:  $[6(x-(x-2) + (x-3))=y$

$Y(x-1) + y(x-2) \dots$

Laurel suggested to use:  $(x)(x-1)(x-2)$

And gave the following example.

When  $x=3$

R	- G	- Y
	- Y	- G
G	- R	-
	- Y	
Y	- R	
	- G	

G4 by Laurel and Sam

0---0----0---0

$$x(x-1)^3$$

3 colors gave 24 ways and that is also true by the above formula

Start with X colors and every other vertex can be any other color but the one before it.  
Which implies one x and three (x-1).

Amber tried to use the method that Laurel gave with G2.

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Cynthia

Cynthia conjectured that the formula for the chromatic polynomial would be  $x(x-1)^n$ , where  $x = \#$  of different colors and  $n = \#$  of edges.

0----0----0----0 with 2 colors is 2, and 4 colors is 108.

Cynthia tried to make the argument more visual by using clothes and accessories as options.

Le had a Quadrilateral

2 gave 2 possibilities

3 colors gave 18 possibilities.

4 colors gave 72 possibilities

But she didn't end up with an equation.