

Documentations of Parameterizations of All Knots Six Crossings or Less and Identification of Brown's Equations

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September 17, 2006

1 Introduction

All topological knots of six crossing or less have been parameterized, some more recently than others. Using a program similar to Maple or Mathematica and a good knowledge of how polynomials behave, it is possible to graph a three dementional parameterization.

2 Parameterizations

Each of the following parameterizations have been compiled from *Example of Polynomial Knots*, written by Ashley Brown[1], *Polynomial Knots*, Written by Donal O'Shea and Alan Durfee[2], and the research done by the 2006 Mount Holyoke REU research team, Susan Durst, Emily List, Charlie Siegel and Matthew Wright[3].

The Shastri Trefoil:[1] Though this parameterization was found in Ashley Brown's paper, this particular parameterization was discovered by Anant R. Shastri.

$$\begin{aligned}x(t) &= t^3 - 3t \\y(t) &= t^4 - 4t^2\end{aligned}$$

$$z(t) = t^5 - 10t$$

The 4₁ Knot:[1] This parameterization is also one of Shastri's polynomials.

$$\begin{aligned} x(t) &= t^3 - 5t \\ y(t) &= t^5 - 28t \\ z(t) &= t^7 - 32t^3 \end{aligned}$$

The 5₁ Knot:[1]

$$\begin{aligned} x(t) &= t^4 - 24t^2 \\ y(t) &= t^5 - 36t^3 + 260t \\ z(t) &= t^7 - 31t^5 - 168t^3 + 560t \end{aligned}$$

The 5₂ Knot:[3]

$$\begin{aligned} x(t) &= t^4 - 12t^2 \\ y(t) &= t(t^2 - 4)(t^2 - 11) \\ z(t) &= (t - .5)(t - 10)(t - 12.5)(t - 20) \cdot \\ &\quad (t - 28)(t - 29)(t - 40)(t - 50.2)(t - 50.6) \end{aligned}$$

The 6₁ Knot:[3]

$$\begin{aligned}
x(t) &= (t^2 - .5)(t^2 - 15.3) \\
y(t) &= t(t^2 - 16)(t^2 - 7)(t^2 - 4.5) \\
z(t) &= t(t^2 - 15.2)(t^2 - 9)(t^2 - 6.25)(t^2 - 1)(t^2 - .25)
\end{aligned}$$

The 6₂ Knot:[1]

$$\begin{aligned}
x(t) &= t^4 - 12t^2 \\
y(t) &= t(t^2 - 4)(t^2 - 11) \\
z(t) &= t(t^2 - 1)(t^2 - 9)\left(t^2 - \frac{49}{16}\right)\left(t^2 - \frac{169}{16}\right)\left(t^2 - \frac{100}{9}\right)
\end{aligned}$$

The 6₃ Knot:[2]

$$\begin{aligned}
x(t) &= t^3 - 100t \\
y(t) &= t(t^2 + 4)(t - 6)(t - 8)(t - 9)(t + 9)(t - 11)(t + 11) \\
z(t) &= (t + 1)(t^2 - 4)(t - 6)(t + 7)(t + 9.2)(t - 9.5)(t + 10) \cdot \\
&\quad (t - 10.5)(t + 11.5)
\end{aligned}$$

3 Identifications

Ashley Brown's paper on polynomial knots gave many examples of polynomial parameterizations, however they were unindented. The following is a list of those knots, identified, using a variety of mathematical methods, including physical construction and Mathematica or Maple plotting.

The 5₁ Knot:[1]

$$\begin{aligned}
x(t) &= t^4 - 24t^2 \\
y(t) &= t^5 - 36t^3 + 260t \\
z(t) &= t^7 - 31t^5 - 168t^3 + 560t
\end{aligned}$$

The 6₂ Knot:[1]

$$\begin{aligned}
x(t) &= t^4 - 12t^2 \\
y(t) &= t(t^2 - 4)(t^2 - 11) \\
z(t) &= t(t^2 - 1)(t^2 - 9)(t^2 - \frac{49}{16})(t^2 - \frac{169}{16})(t^2 - \frac{100}{9})
\end{aligned}$$

The 7₄ Knot:[1]

$$\begin{aligned}
x(t) &= t(t^2 - 6)(t^2 - 12) \\
y(t) &= t(t^2 - 7)(t^2 - 9) \\
z(t) &= t(t^2 - 1)(t^2 - 9)(t^2 - \frac{1}{16})(t^2 - \frac{81}{16})(t^2 - \frac{25}{4})(t^2 - \frac{256}{25})
\end{aligned}$$

The 8₂ Knot:[1]

$$\begin{aligned}
x(t) &= t^5(t^2 - 8)(t^2 - 10) \\
y(t) &= t^4 - 11t^2 \\
z(t) &= t(t^2 - 1)(t^2 - 4)(t^2 - 9)(t^2 - \frac{25}{16})(t^2 - \frac{961}{100})(t^2 - \frac{3989}{400})(t^2 - \frac{361}{36})
\end{aligned}$$

The 10₁₁₆ Knot:[1]

$$\begin{aligned}x(t) &= t(t^2 - 4)(t^2 - 9) \\y(t) &= t^2(t^2 - 7)(t^2 - 8) \\z(t) &= t(t^2 - 1)\left(t^2 - \frac{1}{16}\right)\left(t^2 - \frac{9}{4}\right)\left(t^2 - \frac{441}{100}\right)\left(t^2 - \frac{81}{16}\right)\left(t^2 - \frac{25}{4}\right) \cdot \\&\quad \left(t^2 - \frac{841}{100}\right)\left(t^2 - \frac{3721}{400}\right)\left(t^2 - \frac{961}{100}\right)\end{aligned}$$