1. A tank holds 500 gallons of pure water. A solution containing 2 lbs of salt per gallon enters the tank at a rate of 3 gal/min. A drain is opened in the bottom of the tank so that the volume of the tank stays constant. Let $S(t)$ be the amount of salt in the tank at time $t$, where $t$ is measured in minutes and $S(t)$ is measured in pounds.

(a) Write the DE that models the rate of change in the amount of salt in the tank $\frac{dS}{dt}$.
(b) The equation you get should be separable. Find the general solution of the DE using the technique of separation of variables.
(c) The equation can also be considered a linear equation. Solve it again using our latest techniques: First, find the general solution to the associated homogeneous equation; second, guess a particular solution to the nonhomogeneous equation; third, use the extended linearity principle to get the final general solution.
(d) Now find the exact solution to the DE with initial condition $S(0) = 0$.

2. A 500 gallon tank is filled with 250 gallons of pure water. A solution containing 2 lbs of salt per gallon enters the tank at a rate of 3 gal/min. A drain is opened in the bottom of the tank allowing the well-mixed solution to leave the tank at a rate of 1 gal/min. Let $S(t)$ be the amount of salt in the tank at time $t$, where $t$ is measured in minutes and $S(t)$ is measured in pounds.

(a) Write the DE that models the rate of change of the amount of salt in the tank. Note that the amount of salt going in is the same as in problem 1. But the amount of salt coming out will have a different formula because the volume is not constant.

The amount of salt coming out is the salt concentration times 1 gal/min. Salt concentration is (amount of salt) divided by (volume of tank). As before the amount of salt is simply $S(t)$. What is $V(t)$, the volume of the tank at time $t$?
(b) You should end up with a linear DE. Find the general solution to the associated homogeneous equation.
(c) Guess a solution to the particular non homogeneous equation. You might have to guess a couple of times before you get it right.
(d) Now use the extended linearity principle to get the general solution to the linear DE, then find the solution to the DE with initial conditions $S(0) = 0$.
(e) The equation can also be solved using an integrating factor. Solve the IVP with initial condition $S(0) = 0$ again, using an integrating factor.
Groups for Class. (You can work with anyone when you’re finishing this for homework.)

When you get together in your group, make sure you learn everyone’s name!

I.

II.

III.

IV.

V.