Algorithm Design

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Course Goals

- Learn how to formulate precise problem descriptions
- Learn specific algorithm design techniques and how to apply them
- Learn how to analyze algorithms for efficiency and for correctness
- Learn when no exact, efficient solution is possible

Stable Matching Problem

Goal. Given n colleges and n students, find a suitable matching.
Colleges rank students in order of preference.
Students rank colleges.

- Each college gets exactly one student.
- Each student gets exactly one college.

Stable Matching Problem

- Goal. Given a set of preferences among colleges and high school students, design an admissions process with these properties:
- Perfect matching: everyone is matched one-to-one.
  - Each college gets exactly one student.
  - Each student gets exactly one college.
- Stability: no incentive for some pair of participants to undermine assignment by joint action.
  - In matching M, an unmatched pair c-s is unstable if college c and student s prefer each other to current partners.
  - Unstable pair c-s could each improve by swapping with current assignments.
- Stable matching: perfect matching with no unstable pairs.

Question 1

- Do all problems have a stable matching?

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is there a stable matching?

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–B, C–D a stable matching?

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–B, C–D a stable matching?

No! B–C is an instability

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–C, B–D a stable matching?

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
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	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–C, B–D a stable matching?

No! A–B is an instability

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
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Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–D, B–C a stable matching?

Stable Roommate Problem

- Goal. Given 2n students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is A–D, B–C a stable matching?

No! A–C is an instability

More Questions

- If the sets being matched are disjoint, is there always a stable matching?
- Is the stable matching always unique?
- Can we find a stable matching efficiently?

Thoughts on Solving the Problem

- Initially, no colleges and students are matched.
- Pick an arbitrary college and have it admit its favorite student. Are we guaranteed that pair will be part of a stable matching?
  - Should a student accept her first offer?
  - If not, what should the student do?
- When are we done? Do we need to consider all combinations???

Propose-and-Reject (Gale-Shapley) Algorithm

```
Initialize each college and student to be free.
while (some college is free and hasn't accepted
    every student) {
    Choose such a college c
    s = 1st student on c's list that c has not
        yet accepted
    if (s is free)
        assign c and s to each other
    else if (s prefers c to her current college
             c')
        assign s and c to each other, and c' to
           be free
    else
        s rejects c
}
```


Questions about the Gale-Shapley Algorithm

- Does the loop terminate?
- Is the matching perfect, that is, is it one-to-one?
- Is the matching stable?

Proof by Contradiction (Review)

- How do we construct a "proof by contradiction"?

1. Assume the thing we want to prove is false.
2. Reason to a contradiction.
3. Conclude that it must therefore be true.