

# Algorithm Design

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## Course Goals

- Learn how to formulate precise problem descriptions
- Learn specific algorithm design techniques and how to apply them
- Learn how to analyze algorithms for efficiency and for correctness
- Learn when no exact, efficient solution is possible

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# Stable Matching Problem

Goal. Given  $n$  colleges and  $n$  students, find a "suitable" matching.  
Colleges rank students in order of preference.  
Students rank colleges.

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# Stable Matching Problem

- Goal. Given a set of preferences among colleges and high school students, design an admissions process with these properties:
- Perfect matching: everyone is matched one-to-one.
  - Each college gets exactly one student.
  - Each student gets exactly one college.
- Stability: no incentive for some pair of participants to undermine assignment by joint action.
- In matching  $M$ , an unmatched pair  $c-s$  is unstable if college  $c$  and student  $s$  prefer each other to current partners.
- Unstable pair  $c-s$  could each improve by swapping with current assignments.
- Stable matching: perfect matching with no unstable pairs.

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## Question 1

- Do all problems have a stable matching?

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## Stable Roommate Problem

- Goal. Given  $2n$  students, find a "suitable" matching.
- Students rank each other.

	Preferences		
Angel	Brat	Crook	Doofus
Brat	Crook	Angel	Doofus
Crook	Angel	Brat	Doofus
Doofus	Angel	Brat	Crook

Is there a stable matching?

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## More Questions

- ⦿ If the sets being matched are disjoint, is there always a stable matching?
- ⦿ Is the stable matching always unique?
- ⦿ Can we find a stable matching efficiently?

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## Thoughts on Solving the Problem

- ⦿ Initially, no colleges and students are matched.
- ⦿ Pick an arbitrary college and have it admit its favorite student. Are we guaranteed that pair will be part of a stable matching?
  - ⦿ Should a student accept her first offer?
  - ⦿ If not, what should the student do?
- ⦿ When are we done? Do we need to consider all combinations???

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# Propose-and-Reject (Gale-Shapley) Algorithm

```
Initialize each college and student to be free.
while (some college is free and hasn't accepted
every student) {
  Choose such a college c
  s = 1st student on c's list that c has not
  yet accepted
  if (s is free)
    assign c and s to each other
  else if (s prefers c to her current college
  c')
    assign s and c to each other, and c' to
    be free
  else
    s rejects c
}
```

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## Question

- For a given problem instance, there may be several stable matchings. Do all executions of *Gale-Shapley* yield the same stable matching, even if we pick colleges in a different order? If so, which one?

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## Question

- ⦿ For a given problem instance, there may be several stable matchings. Do all executions of Gale-Shapley yield the same stable matching? If so, which one?

Def. College  $c$  is a valid partner of student  $s$  if there exists some stable matching in which they are matched.

College-optimal assignment. Each college receives best valid student.

Claim. All executions of *GS* yield college-optimal assignment, which is a stable matching!

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## Algorithm Design

- ⦿ Formulate the problem precisely
- ⦿ Design an algorithm to solve the problem
- ⦿ Prove the algorithm correct
- ⦿ Analyze the algorithm's runtime (Come back next time...)

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