Lie Groups: A Problem-Oriented Introduction via Matrix Groups
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MAA Textbooks 2009

These are corrections known to me as of May 10, 2010. I am grateful to my spring 2010 students at Mount Holyoke and also to Emily Moore (Grinnell College) and David Murphy (Hillsdale College) for some of them. Notice of other errors or omissions will be much appreciated; please email hpollats@mtholyoke.edu.

The note *About the cover* should refer to Problem 5.1.10.

Page 11: The definition of $O(\mathbb{R}^3)$ should not included invertibility. It should be

$$\{T : \mathbb{R}^3 \to \mathbb{R}^3 \ : T \text{ linear transformation and } T \text{ preserves distance}\}.$$

(This error is repeated three times, including in Problem 1.3.5, part (e) of which is to deduce from the definition that T is invertible.)

Page 13, line 12: It should be I_n, not I_3.

Page 13, line 14: Problem 1.3.8 should be in bold.

Page 17 The definitions of $\mathcal{L}^+(\mathbb{R}^4)$ and $\mathcal{L}^+(2, \mathbb{R})$ should be added (each referring to the elements of the corresponding Lorentz group of determinant 1).

Page 18, line 4: the reference should be to (1.4.6f).

Page 19, Problem 1.4.14: The student should assume that the determinant of the block matrix

$$\begin{bmatrix} A & 0_2 \\ 0_2 & B \end{bmatrix}$$

is $\det(A)\det(B)$.

Page 23, line -9: The geometric description of the reflection is more natural for $0 \leq \alpha < 2\pi$.

Page 55, Problem 3.2.13: The (1,2) entry of the matrix $F(q)$ should be $-c - di$, and the (2,1) entry should be $c - di$. (Else the function F is not a homomorphism.)

Page 77, Problem 4.4.5: David Murphy has pointed out that curves "crossing" at a point ought to mean not only that they have a common point but also that at the common point they have different tangents. In other words, Artin's problem is asking whether a one-parameter subgroup γ in a matrix group can satisfy both $\gamma(s) = \gamma(t)$ and also $\gamma'(s) \neq \gamma'(t)$ for some $s, t \in \mathbb{R}$.

Page 79, Problem 4.5.3c: should refer to (b), not (c).

Page 86, Problem 4.6.12c: The instructions should say to include examples with $x_1^2 < x_2^2$, $x_1^2 = x_2^2$ and $x_1^2 > x_2^2$ rather than referring to absolute values. (And the solution in the Instructor's Manual should do the same.)

Page 104, Problem 5.1.14d should say show $[T, S] = I$ (since $[S, T] = -I$).

Page 107, Problem 5.2.11a actually doesn't require Theorem 4.4*. (Thanks to my student Dana Botesteanu who found this proof.)

Page 109: There is a relatively elementary proof of Theorem 5.2*. It requires the following

Lemma Assume V is a finite dimensional real vector space and $f : \mathbb{R} \to V$ is differentiable. Then $f'(0) \in V$.

The lemma follows from the assumption that V is closed, so that difference quotients $(1/h)(f(h) - f(0)) \in V$ for all nonzero $h \in \mathbb{R}$ imply $f'(0) \in V$.

Now the proof of Theorem 5.2* is as follows. Choose A, B in $L(G)$ with $A = \alpha'(0)$ and $B = \beta'(0)$ for smooth functions α, β from $\mathbb{R}$ to G with $\alpha(0) = \beta(0) = I_n$. Let $\gamma_s(t) = \alpha(s)\beta(t)\alpha(-s)$. Then for all s, $\gamma_s(t)$ is in G for all t, and $\gamma_s(0) = I_n$. Therfore $f(s) = \gamma_s'(0) = \alpha(s)B\alpha(-s) \in L(G)$. Apply the lemma to conclude that $f'(0) = AB - BA$ is in $L(G)$.

Page 113: For Family C, the block matrix describing a typical family member should have $-M_{11}^T$ in the lower right corner.

Page 120, Problem 6.2.8f requires the results of (6.2.11) and (6.2.12).

Page 121, Problem 6.2.10d should refer to parts (b) and (c).

Page 121, Problem 6.2.13 should include the assumptions in (6.2.10).

Page 125, Problem 6.3.9d should refer to (6.3.5b).