Lie Groups: A Problem-Oriented Introduction via Matrix Groups
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These are corrections known to me as of April 17, 2013. I am grateful to my spring 2010 students at Mount Holyoke and also to Emily Moore (Grinnell College) and David Murphy (Hillsdale College) for some of them. Julie Beier (Mercer University) and her students called attention to the complexity of Problem 1.3.11(b). Giuliano Gnugnoli noted the unclear statement on pages 26/27. Notice of other errors or omissions will be much appreciated; please email hpollats@mtholyoke.edu.

The note *About the cover* should refer to Problem 5.1.10.

Page 11: The definition of $O(\mathbb{R}^3)$ should not included invertibility. It should be

$$\{T : \mathbb{R}^3 \to \mathbb{R}^3 \ : T \text{ linear transformation and } T \text{ preserves distance}\}.$$

(This error is repeated three times, including in Problem 1.3.5, part (e) of which is to deduce from the definition that T is invertible.)

Page 13, line 12: It should be I_n, not I_3.

Page 13, line 14: Problem 1.3.8 should be in bold.

Page 14, line 13: Problem 1.3.11 should be revised. The current part (c) should be part (b). The new part (c) should say: Investigate whether your conditions in (a) are necessary as well as sufficient, using examples or arguments as appropriate. For the special case of 2×2 matrices, try to sort out exactly what happens when you weaken your condition in (a). Hint: Define a relation $\sim$ on $m \times m$ matrices by $C \sim C'$ if there exists an invertible matrix M with $C' = M^T C M$. Show this is an equivalence relation, and show that if one of two equivalent matrices defines a group, then so does the other.

Page 17 The definitions of $\mathcal{L}^+(\mathbb{R}^4)$ and $\mathcal{L}^+(2, \mathbb{R})$ should be added (each referring to the elements of the corresponding Lorentz group of determinant 1).

Page 18, line 4: the reference should be to (1.4.6f).

Page 19, Problem 1.4.14: The student should assume that the determinant of the block matrix

$$\begin{bmatrix} A & 0_2 \\ 0_2 & B \end{bmatrix}$$

is $\det(A)\det(B)$.

Page 23, line -9: The geometric description of the reflection is more natural for $0 \leq \alpha < 2\pi$.

Pages 26/27: The statement about which subgroups of $GL(n,\mathbb{R})$ are Lie groups is not sufficiently clear. A matrix Lie group doesn't have to be closed in $\mathcal{M}(n,\mathbb{R})$. (In particular, $GL(n,\mathbb{R})$ itself isn't closed: consider the sequence of diagonal matrices $D_m = \text{diag}(1/m, 1/m, \ldots, 1/m)$ for $m \geq 1$, with limit equal to the zero matrix.) However, in order to be a Lie group, a subgroup $H \subseteq GL(n,\mathbb{R})$ has to be closed in the following sense: if a sequence A_m, $m \geq 1$, of matrices in H has limit $A \in GL(n,\mathbb{R})$, then A must be an element of H. In other words, either A must be in H or A must be singular.

Page 55, Problem 3.2.13: The (1,2) entry of the matrix $F(q)$ should be $-c - di$, and the (2,1) entry should be $c - di$. (Else the function F is not a homomorphism.)

Page 77, Problem 4.4.5: David Murphy has pointed out that curves "crossing" at a point ought to mean not only that they have a common point but also that at the common point they have different tangents. In other words, Artin's problem is asking whether a one-parameter subgroup γ in a matrix group can satisfy both $\gamma(s) = \gamma(t)$ and also $\gamma'(s) \neq \gamma'(t)$ for some $s, t \in \mathbb{R}$.

Page 79, Problem 4.5.3c: should refer to (b), not (c).

Page 86, Problem 4.6.12c: The instructions should say to include examples with $x_1^2 < x_2^2$, $x_1^2 = x_2^2$ and $x_1^2 > x_2^2$ rather than referring to absolute values. (And the solution in the Instructor's Manual should do the same.)

Page 104, Problem 5.1.14d should say show $[T, S] = I$ (since $[S, T] = -I$).

Page 107, Problem 5.2.11a actually doesn't require Theorem 4.4*. (Thanks to my student Dana Botesteanu who found this proof.)

Page 113: For Family C, the block matrix describing a typical family member should have $-M_{11}^T$ in the lower right corner.

Page 120, Problem 6.2.8f requires the results of (6.2.11) and (6.2.12).

Page 121, Problem 6.2.10d should refer to parts (b) and (c).

Page 121, for Problem 6.2.13, the assumptions in (6.2.10) apply.

Page 125, Problem 6.3.9d should refer to (6.3.5b).