Edge-Disjoint Spanning Trees and Inductive Constructions

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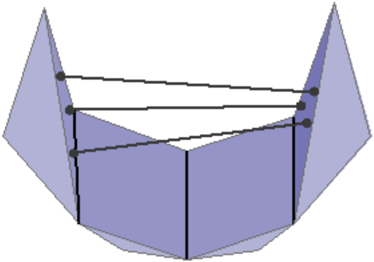
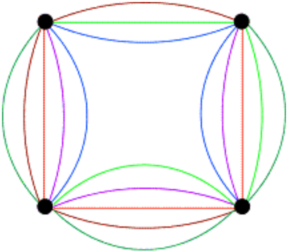


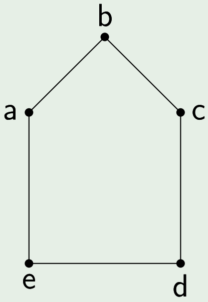


A 3D body-and-bar structure is rigid $\Leftrightarrow$ the associated graph is the union of 6 edge-disjoint spanning trees. [Tay,1984]

Definitions

A **cycle** is a graph with an equal number of vertices and edges whose vertices may be placed around a circle in such a way that two vertices are adjacent if and only if they appear consecutively along the circle. A graph with no cycle is called **acyclic**.

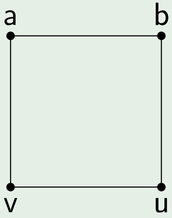
Example



Definitions

A graph G is **connected** if for all $u, v \in V(G)$ there exists a path from u to v.

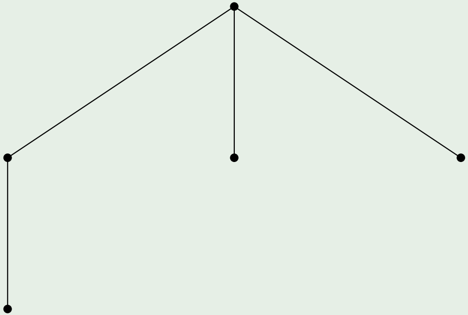
Example



Definitions

A graph G is a **tree** if it is connected and acyclic.

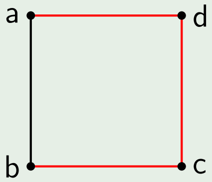
Example



Definitions

A tree $T = (V_T, E_T)$ is a **spanning tree** of a graph $G = (V, E)$ if $V_T = V$.

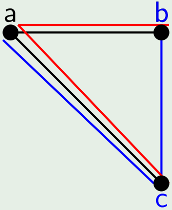
Example



Definition

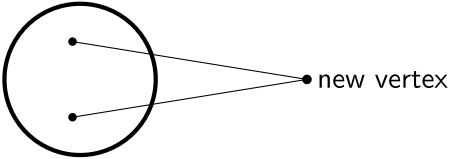
Let $G = (V, E)$ be a multigraph with no loops.
G is (k, k)-tight if there is a partition of the edge set such that there are k edge-disjoint spanning trees.

Example



Previous Work

- 2D Rigidity, Henneberg 0 and 1 [Henneberg 1911] [Laman 1970]

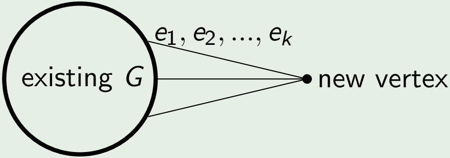


- Body-and-Bar Rigidity using Henneberg operations inductively to form tight graphs [Tay 1991]
- Generalized Counting Characterizations [Haas 2002]

Inductive constructions are prevalent in Rigidity Theory because they are often useful in proofs.

Henneberg 0: Inductive construction builds a rigid structure one vertex at a time.

Example



$$\text{graph } G = (V, E)$$
$$\text{new graph } G' = (V \cup v, E \cup e_1, e_2, ..., e_k)$$

This manner of inductive construction preserves the property of k edge-disjoint spanning trees.

single vertex

$k = 3$

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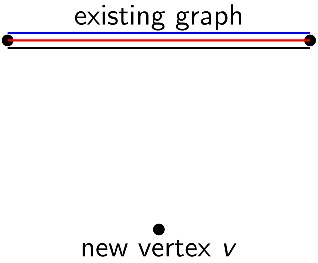
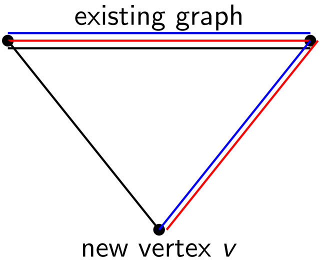
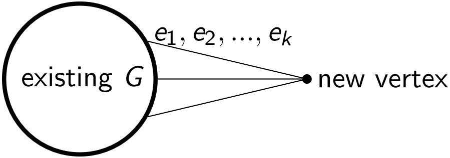


$k = 3$



$k = 3$

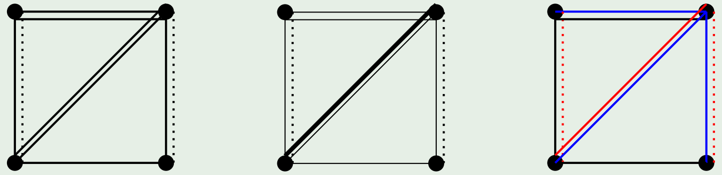


graph $G = (V, E)$
new graph $G' =$
$(V \cup v, E \cup e_1, e_2, ..., e_k)$

Body-and-CAD Rigidity- recent results for 2D

- Combinatorial Characteristics of 2D Body-and-CAD Rigidity: associated graph $G = (V, S \sqcup D)$, exists $S' \subseteq S$ such that $S \setminus S'$ is edge-disjoint union of 2 spanning trees and $D \cup S'$ is edge-disjoint union of 1 spanning tree [Sidman,Lee-St. John,LaMon 2012]

Example



- We study a new class of graphs that arise from rigidity of body-and-CAD structures.

Body-and-CAD Rigidity- recent results for 3D

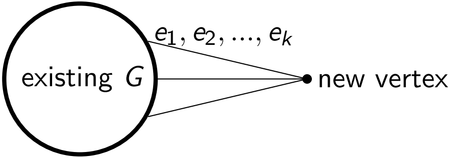
- Combinatorial Characteristics of 3D Body-and-CAD Rigidity: associated graph $G = (V, S \sqcup D)$, exists $S' \subseteq S$ such that $S \setminus S'$ is edge-disjoint union of 3 spanning trees and $D \cup S'$ is edge-disjoint union of 3 spanning tree [Sidman,Lee-St. John 2012]
- this does not include point-point coincidence constraints

Generalizing Counts

- 2D: $2 + 1 = 3$
- 3D: $3 + 3 = 6$
- general: Let $a + g = k$, where a is the number of solid trees, g is the number of trees that might include dashed edges, and k is the total number of spanning trees.

Generalizing the Construction

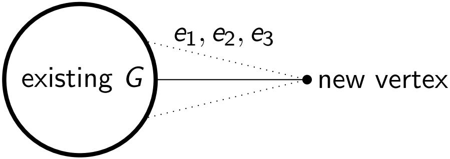
- can partition into $k = a + g$ spanning trees
- inductive constructions for (k, k)-tight graphs exist
- generalize inductive construction to take into account dashed and solid edges



graph $G = (V, S \sqcup D)$

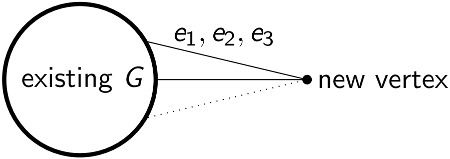
new graph $G' = (V \cup v, S \cup e_1, \ldots, e_n \sqcup D \cup e_{n+1}, \ldots, e_k)$

$k = 3, g = 2$



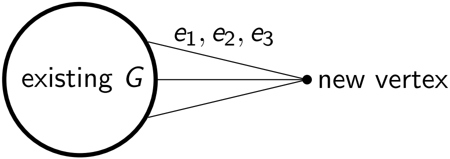
graph $G = (V, S \sqcup D)$
new graph $G' = (V \cup v, S \cup e_2 \sqcup D \cup e_1 \cup e_3)$

Also $k = 3, g = 2$



graph $G = (V, S \sqcup D)$
new graph $G' = (V \cup v, S \cup e_1 \cup e_2 \sqcup D \cup e_3)$

Or even $k = 3, g = 2$

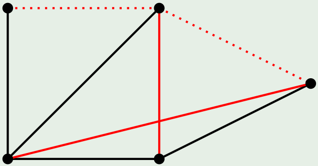


graph $G = (V, S \sqcup D)$
new graph $G' = (V \cup v, S \cup e_1 \cup e_2 \cup e_3 \sqcup D)$

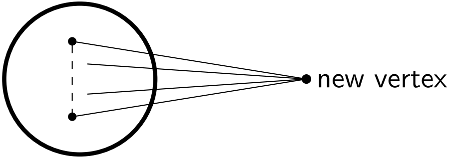
This implies that this constructive step creates a new graph that satisfies the $k = a + g$ spanning trees property.

There are some (k, k)-tight graphs which my constructive operation cannot form.

Example



Since my inductive construction does not create every (k, k)-tight graph, I would like to modify my construction to also use the other Henneberg move which removes edges before adding the new vertex.



Questions?