

Problem Set

Recall our model of hidden action from class.

Model

- Level of output x . Restrict to $x \in \{x_H, x_L\}$ with $x_H > x_L$.
- Agent chooses level of effort e . Restrict to $e \in \{a, b\}$ where a is high effort and b is low effort.
- Distribution of x depends on e : In our case $\pi_a = \text{Prob}[x = x_H | e = a], \pi_b = \text{Prob}[x = x_H | e = b]$, $\pi_a > \pi_b$.
- The principal pays the agent a salary $s(x)$ that can depend on the level of output realized. Here we have s_H, s_L .
- Agent pays cost of effort $c(e)$. Here, let $c(a) = C, c(b) = 0$.
- Principal has utility $x - s$. Linear utility implies risk-neutral.
- Agent has utility $u(s) - c(e)$ where $u' \geq 0$ and $u'' \leq 0$. Concave utility function implies risk aversion.
- Agent has reservation utility \bar{u} .

Timing

1. Principal chooses contract s_H, s_L .
2. Agent chooses level of effort e .
3. Output x is realized and utility received.

Problems

1. 25 points
In the full information case where the principal can directly observe the action of the agent, we solved for the optimal contract that induces the agent to choose effort level a . Now, find the optimal contract that induces the agent to choose $e = b$.
2. 25 points
In the hidden action case we proved that both the (IR) and (IC) constraints bind at the optimal contract. With this information solve the system of two equations given by the constraints to find s_H^* and s_L^* .
3. 15 points
Compare the optimal contracts that induce the agent to choose $e = a$ in the full information and hidden information case. Is the expected utility to the agent greater, less, or equal in the hidden action case compared to the full information case?
4. 35 points
Let $x_H = 18, x_L = 1, \pi_a = 2/3, \pi_b = 1/3, C = 0.1, u(s) = s^{1/2}, \bar{u} = 0.1$.
 - (a) What is the first best contract when actions are observable?
 - (b) What is the optimal contract when actions are not observable?
 - (c) Suppose the principal could pay M in order to observe the agent's action. What is the largest M the principal would be willing to pay?