Corporate Board Dynamics: Directors Voting for Directors

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Job Market Paper
Fall 2008

Abstract

I propose a dynamic model in which corporate directors perform firm tasks (such as monitoring management) and choose new directors. Previous literature has focused on the board composition that statically optimizes firm tasks. I incorporate features of these models and give directors the additional task of hiring new directors. This introduces an important dynamic element: the board must consider both how new directors will perform in firm tasks and how new directors will try to change board composition in future hiring rounds. I find that the optimal board composition in the dynamic model differs from that of the static model. Additionally, lack of a commitment mechanism means directors do not always choose board compositions that maximize shareholder value. This creates an opportunity for policy to benefit shareholders. In 2003, the NYSE and NASDAQ exchanges implemented two new rules. First, boards must be composed of a majority of outside directors. Second, director selection must be done by a nominating committee composed of outside directors. I use the model to analytically and numerically investigate the effects of these new regulations on shareholder value. I find that the regulations may benefit shareholders of a firm in the dynamic environment, but never benefit shareholders in the static environment.

JEL Classification: G3, L5, D7

Keywords: board of directors, voting for voters, corporate governance

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*I would like to thank my advisor Thomas Holmes for his continuous support. I also thank Erzo Luttmer, Jan Werner, Rajesh Aggarwal and participants of the Applied Micro Workshop at the University of Minnesota. All errors are mine.

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1 Introduction

The board of directors performs many functions for a firm. Primarily, the board acts on behalf of shareholders to monitor management and ensure that those in control of the firm are taking actions that maximize shareholder value. In addition to monitoring, the board assists management in long term strategic decision making. Both of these duties are “firm tasks” – the outcomes directly affect the firm and shareholder value.

Directors perform another task related to the board itself: choosing new directors. Formally, shareholders choose new directors in annual elections. In practice, incumbent directors almost always nominate one candidate and shareholders vote in an uncontested election. The institutional details leading to this arrangement are discussed below. The assumption that directors choose new directors leads to several dynamic considerations that have not been studied in previous literature and are the focus of this paper. First, once the firm goes public, shareholders no longer have control of the board’s composition. When shareholders set up the initial board, they must consider both how the directors will perform in firm tasks and how the directors will change the board composition once control is handed over. If directors cannot commit to maintain a particular board composition, shareholders may choose an initial composition that does not maximize the value of firm tasks. Second, once the board is in control, incumbent directors face these same commitment problems when choosing new directors.

Directors are commonly grouped into two types: independent outsiders that have no financial ties to the firm (other than director remuneration) and insiders that lack this financial independence.\footnote{For formal definitions, see NYSE Listed Company Manual, section 303A.02 and Rule 4200(a)(15) of the NASDAQ exchange.} In the dynamic setting, a board may find it myopically optimal to hire one additional insider to maximize the value of firm tasks, but that insider may try to hire additional insiders in the future.

As another example, consider a “management friendly” board composed of inside directors. Because of their financial ties to management, the inside directors have incentive to not monitor and will not hire outside directors that will enforce monitoring. The board is then entrenched...
in an insider dominated state, potentially at the detriment of shareholder value. Instances of lax boards not monitoring management led to several recent regulatory changes. In 2003, the NYSE and NASDAQ stock exchanges implemented two new regulations. First, boards of member firms must be composed of a majority of independent outside directors. This rule acts as a commitment mechanism by not allowing directors to take the board into a majority insider state. Second, the NYSE requires the board to have a nominating committee composed of independent outside directors for the purpose of choosing new directors. This rule targets the director selection process and delegates control to outsiders, thus eliminating the concern that inside directors will hire additional insiders.

In this paper I propose a dynamic stochastic game in which directors perform firm tasks and choose new directors. Before the game begins, shareholders set initial conditions by choosing the first board composition. Each period is split into two stages. First, directors perform firm tasks in a simple static game. Second, anticipating that one director will retire at the end of the period, the board receives an inside and outside candidate to fill the open seat. Besides differing in their insider/outsider designation, candidates also differ in quality. The incumbent directors choose one of the candidates to join the board through majority voting. The combination of the type of director that retires and the type of the newly hired director determines next period’s board composition.

I find that the board composition that optimizes shareholder value in the static model may not be the same as the composition that maximizes shareholder value in the dynamic model. As a result, shareholders will choose different initial board compositions in the static and dynamic models. Additionally, directors are not perfectly aligned with shareholders and do not always choose board compositions that maximize shareholder value. This creates an opportunity for regulation to increase shareholder value. In the static model, the majority outsider regulation and the nominating committee regulation never increase shareholder value and may even decrease it. In the dynamic model, both rules have the potential to benefit shareholders. The rules are not perfect though. The majority outsider regulation helps solve a commitment problem, but at the cost of reducing the flexibility of the board in responding to short run needs of the firm. The nominating committee regulation grants director selection rights to outsiders that are more closely aligned to shareholders
than insiders. But when outsiders are not perfectly aligned with shareholders, the regulation may remove a barrier that prevents outsiders from hiring inside candidates that shareholders do not want.

This paper lies in the intersection of several areas of current research. First, it contributes to the literature on corporate governance and the endogenous determination of corporate boards. Second, it contributes to the small but growing literature on dynamic stability in clubs.

Many papers theoretically model the board of directors and explore the effects of board composition on shareholder value. Harris and Raviv (2008), Adams and Ferreira (2007), Raheja (2005) and Hermalin and Weisbach (1998) all explore how board composition affects the outcome of firm tasks. A common theme in these papers is that the optimal board includes both insiders and outsiders and the specific composition depends on firm and environment specific parameters. Unlike these papers, in my model I do not provide a rich micro-foundation of firm tasks. Instead, I include firm tasks as a static game and focus on the dynamic problem of directors choosing new directors. This paper is closest in nature to Hermalin and Weisbach (1998) in that incumbent directors play a significant role in the selection of new directors. Moving to a dynamic framework allows me to capture important aspects of boards, such as entrenchment and commitment, that do not occur in the environment of Hermalin and Weisbach. In addition, the new regulations discussed here target aspects of the board that are only fully revealed in a dynamic environment.

The idea of voters voting for voters and dynamic stability in clubs has been investigated in a small but growing literature. Roberts (1999) and Barbera et al. (2001) consider environments in which club members vote for new members that participate in future membership votes. These papers put heavier restrictions on preferences and the set of potential members. Acemoglu et al. (2008) also develop this literature with axiomatic and game theoretic approaches. My model allows greater flexibility and stochastic elements at the cost of analytical tractability.

I now address the institutional details of how new directors are chosen. In the model, incumbent directors select new directors. In reality, directors of a public corporation are chosen for a seat on the board via annual elections in which shareholders vote (usually one vote per share owned) on a ballot containing one or more candidates per open seat (Sjostrom Jr, 2007). In practice, several
obstacles prevent this mechanism from effectively transmitting shareholder preferences into the board’s composition. First, director candidates are usually nominated by the incumbent board. Shareholders may nominate candidates via a shareholder proxy, but SEC rules make this difficult.\(^2\) Recent efforts have been made to grant shareholders easier access to the election ballot, but the SEC ruled in 2008 to keep the barriers in place (Securities and Comission, 2008). Second, nearly all candidates run in uncontested elections (Fischer et al., 2008). Paired with the plurality voting mechanism employed by many corporations, it is possible for a candidate to be voted onto the board with only a single affirmative vote. The resulting system is one in which the incumbent board chooses new directors and shareholders have little input in director selection.

The paper continues as follows. Section 2 presents the model. Section 3 presents analytical results from various simplifications of the general model. Section 4 presents numerical results from a more general model and section 5 concludes.

### 2 Model

A shareholder owned firm consists of a board of directors. The board is fixed in size and each director is designated as an insider or an outsider. The state of the firm in each period is simply the composition (number of insiders and number of outsiders) of the board. When taking a firm public, shareholders set initial conditions by choosing the initial composition of the board. After that, shareholders have no further role in managing the firm: all decisions are made by the board. Each period is divided into two stages. First, in the work stage, directors perform firm tasks (monitor management, make policy decisions) that generate payoffs to both the shareholders and directors. These payoffs depend on the composition of the board. Second, directors choose new directors. Unlike shareholders, directors are not infinitely lived: each period one director is selected for retirement and needs to be replaced. Anticipating that a seat will open up at the end of the period, the board receives one inside and one outside candidate and chooses one to fill the open seat. Each incumbent votes for the candidate they prefer and a final decision is made through

\(^2\) Rule 14a-8(i)(8) under the Securities Exchange Act of 1934 permits exclusion of shareholder proposals related to the election of directors.
majority voting. The combination of the type of the replacement director and the type of the retiring director determines next period’s board composition.

The model is set in discrete time with an infinite horizon. All agents discount at a common factor \( \beta \in [0, 1) \). There is one firm consisting of a board of directors of fixed size \( N \). An individual director \( i \) may be in one of two states: \( \omega_i \in \Omega = \{ I, O \} \), corresponding to whether the director is an insider or outsider and a director’s state may not change over time. A board of size \( N \) is then characterized by the list of states of the individual directors: \( \omega = (\omega_1, ..., \omega_N) \). I focus on symmetric and anonymous strategies with a fixed board size, so at any period \( t \) it is sufficient for a director to know his own type and the number of insiders on the board. Let \( s_t \in S = \{0, ..., N\} \) be the number of insiders on the board at time \( t \). For notational simplicity, I suppress time subscripts for the remainder of the paper.

2.1 Work stage

In the work stage, the board performs firm tasks. I keep the work stage specification simple in order to focus most analysis on the dynamic decision of replacing directors. I model the work stage as a simple static game among directors in which all Nash equilibria generate identical payoff profiles to directors and the firm. With board \( s \), the game pays \( w_S(s) \) to shareholders, \( w_I(s) \) to each inside director and \( w_O(s) \) to each outside director. Actions made in the work stage do not directly affect the distribution of future states of the firm and the \( w(\cdot) \) functions are taken as primitives in the full stochastic dynamic game.

In this section I construct the static game that generates these work stage payoffs. Each director makes a left/right decision by choosing an action \( x_i \in \{ L, R \} \). These individual actions are aggregated to a board level action \( x \in \{ L, R \} \) through the majority rule. In the event of a tie, a fair coin toss determines the outcome. Payoffs are determined by the board level decision, so I abstract from the formal definition of the game and look at how payoffs depend on the number of votes in each direction. Let \( |L| \) denote the number of directors that vote for left and \( |R| \) denote the number that vote for right. The firm is not able to contract on the work stage and agency costs may result in insiders preferring a different outcome than outsiders and shareholders.
Left provides a strictly positive payoff to the firm, while right pays nothing. I allow the payoff of left to depend on the composition of the board through the function $\phi(s) > 0$.

\[
w_S(s) = \begin{cases} 
\phi(s) & \text{if } |L| > |R| \\
0 & \text{if } |L| < |R| \\
\frac{1}{2}\phi(s) & \text{if } |L| = |R|
\end{cases}
\]

(1)

Outside directors' work stage payoffs are perfectly aligned with shareholders:

\[
w_O(s) = w_S(s)
\]

(2)

It is clear that shareholder and outside directors always prefer $x = L$. This is not necessarily the case for insiders. Insiders receive a private benefit $B \geq 0$ when the board chooses $x = R$. This private benefit is a measure of the agency cost present in the firm.

\[
w_I(s) = \begin{cases} 
\phi(s) & \text{if } |L| > |R| \\
B & \text{if } |L| < |R| \\
\frac{1}{2}\phi(s) + \frac{1}{2}B & \text{if } |L| = |R|
\end{cases}
\]

(3)

If $B$ is large enough, insiders will prefer $R$.

While somewhat stylized and mechanical, this specification of work stage payoffs is flexible enough to capture the effects of composition on firm task outcomes as suggested in the literature.

For example, in monitoring the CEO, the decision to go left represents active monitoring and right represents no monitoring. The value of not monitoring will naturally be higher for insiders than outsiders and shareholders, and $B > 0$. If the board chooses to monitor the CEO, it is valuable to have insiders on the board as they have more precise information about the quality of the CEO. This is captured by choosing a $\phi(s)$ that is increasing in $s$. This specification is in accordance with the notion that if the board commits to work (i.e. choose left), then the first best board composition is all insiders. Without commitment to choose left, a board of all insiders will instead
choose to capture the private benefit $B$ and move right. Therefore, the second best composition is a mix such that there are enough outsiders to keep the board monitoring management, but also includes insiders to benefit from their more precise knowledge.

2.2 Director replacement stage

I now turn to the dynamic aspects of the model. Each period, directors perform firms tasks and choose one new director. At the end of the period, an incumbent director is chosen at random for retirement and is replaced by the newly hired director. The outcome of the director replacement decision and retirement draw determine the next period’s board composition.

Each period, the board draws two candidate board members: one insider and one outsider. Besides their designations as an insider or outsider, if hired, a candidate brings a randomly drawn and publicly observed one time quality benefit to the firm and incumbent directors. The candidate of type $k \in \{I, O\}$ brings one time benefit $\nu_k$ drawn i.i.d. across time and candidates from distribution $F_k(\cdot) = N[\mu_k, \sigma^2_k]$ with density functions $f_k(\cdot)$. Let $\nu = (\nu_I, \nu_O)$ be the profile of benefit draws. These public benefits have several interpretations. First, it allows individual directors to differ in quality. Exogenous retirement results in all directors having the same expected tenure on the board and any persistent quality component of a director may be wrapped up into a one time expected benefit. Second, since each hiring round brings exactly one inside and one outside candidate, these benefits also capture any current period idiosyncratic preference the firm has for an insider or outsider that is not captured by the work stage payoffs. For expositional clarity, I will refer to this term as a candidate’s quality.

Each incumbent director $i$ also has a private individual preference $\epsilon_{ik}$ for each candidate. The $\epsilon_{ik}$ are drawn from a continuous distribution $G(\cdot)$ with mean $\nu$, variance $\sigma^2_{\epsilon}$ and are i.i.d. across incumbent directors, candidates and time. Let $\epsilon_i = (\epsilon_{iI}, \epsilon_{iO})$ be director $i$’s profile of individual preference shocks. This shock captures any individual preference a director may have for a candidate from sources outside the model, such as social relations.

After observing the current state of the board $s$, the candidate quality draws $\nu$, and individual shocks $\epsilon_i$ each director $i$ votes for the candidate they prefer. The board level decision is determined
by majority voting among the incumbent directors.

2.3 Equilibrium

Throughout, I use $j$ to denote the type of an incumbent and $k$ to denote the type of a candidate. Let $V_j(s, \nu, \epsilon)$ be the expected present value for a director of type $j \in \{I, O\}$ sitting on board $s$ with candidate draws $\nu$ and individual shocks $\epsilon$. A director of type $j$ will vote for the candidate $k$ that provides a higher expected present value $V^k_j(s, \nu, \epsilon)$. Expectations are unconditional over the i.i.d. $\nu'$ and $\epsilon'$.

$$V^O_I(s, \nu, \epsilon) = w_I(s) + \beta \left( 1 - \frac{1}{N} \right) \left[ \nu_I + \epsilon_I + \frac{s}{N-1} E[V^O_I(s, \nu', \epsilon')] \right] + \left( 1 - \frac{s}{N-1} \right) E[V^O_I(s+1, \nu', \epsilon')]$$

$$V^O_O(s, \nu, \epsilon) = w_O(s) + \beta \left( 1 - \frac{1}{N} \right) \left[ \nu_O + \epsilon_O + \frac{s}{N-1} E[V^O_O(s, \nu', \epsilon')] \right] + \left( 1 - \frac{s}{N-1} \right) E[V^O_O(s+1, \nu', \epsilon')]$$

$$V^I_I(s, \nu, \epsilon) = w_I(s) + \beta \left( 1 - \frac{1}{N} \right) \left[ \nu_I + \epsilon_I + \frac{s-1}{N-1} E[V^I_I(s, \nu', \epsilon')] \right] + \left( 1 - \frac{s-1}{N-1} \right) E[V^I_I(s+1, \nu', \epsilon')]$$

$$V^I_O(s, \nu, \epsilon) = w_I(s) + \beta \left( 1 - \frac{1}{N} \right) \left[ \nu_O + \epsilon_O + \frac{s-1}{N-1} E[V^I_O(s-1, \nu', \epsilon')] \right] + \left( 1 - \frac{s-1}{N-1} \right) E[V^I_I(s, \nu', \epsilon')]$$

If hired, a candidate enters an incumbent’s value function in three places. Consider $V^I_O(s, \nu, \epsilon)$, the value of an outside director when the inside candidate is hired. First, the director gets the work stage payoff $w_O(s)$ that only depends on today’s board composition. With probability $\frac{1}{N}$, the director will retire at the end of the period and receive a payoff of zero. With probability $1 - \frac{1}{N}$, the director continues to next period, which he discounts by $\beta$. Next period, the incumbent gets the new insider’s quality draw $\nu_I$ and the new insider’s individual component $\epsilon_I$. Conditional on the outsider surviving, the probability that an insider retired is $\frac{s}{N-1}$. In this case, the new insider replaces a retiring insider and the board composition remains constant at $s$. With probability $1 - \frac{s}{N-1}$, an outsider retired and was replaced by the new insider. In this case, the board composition changes to $s+1$. The expressions for the other three cases are constructed in a similar manner.

Let $q_j(s, \nu; k)$ be the beliefs of a type $j$ director voting for the type $k$ candidate that the inside
candidate is hired. Then the value function for a director of type $j$ is

$$V_j(s, \nu, \epsilon) = \max_{k \in \{I, O\}} q_j(s, \nu; k) V^I_j(s, \nu, \epsilon) + [1 - q_j(s, \nu; k)] V^O_j(s, \nu, \epsilon)$$  \hspace{1cm} (4)$$

$$h_j(s, \nu, \epsilon) = \argmax_{k \in \{I, O\}} q_j(s, \nu; k) V^I_j(s, \nu, \epsilon) + [1 - q_j(s, \nu; k)] V^O_j(s, \nu, \epsilon)$$  \hspace{1cm} (5)$$

From the functions defined in (5), it is clear that in state $s$, a director of type $j$ will vote for the inside candidate when

$$V^I_j(s, \nu, \epsilon) \geq V^O_j(s, \nu, \epsilon)$$

which is equivalent to

$$(\nu_I + \epsilon_I) - (\nu_O + \epsilon_O) \geq V^O_j(s, 0, 0) - V^I_j(s, 0, 0)$$

Let

$$\hat{\nu}_j(s) = V^O_j(s, 0, 0) - V^I_j(s, 0, 0)$$  \hspace{1cm} (6)$$

be the minimum advantage in quality and individual draws that an inside candidate needs to have over an outside candidate for an incumbent of type $j$ to vote for the inside candidate. These cutoff values are a more convenient and more illustrative way of expressing the policy functions.

Given policy of insiders and outsiders, beliefs for a type $j$ incumbent voting for the type $k$ candidate are constructed as follows. Let $p_j(s, \nu)$ be the probability that a type $j$ incumbent gets $\epsilon$ draws such that he votes for the inside candidate.

$$p_j(s, \nu) = \int [1 - G(\hat{\nu}_j(s) - \nu_I + \nu_O + \epsilon_O)] g(\epsilon_O) d\epsilon_O$$

Then construct the probability $p_j(s, \nu, m)$ that the inside candidate gets $m$ votes from the $N - 1$ directors that remain after removing a type $j$ incumbent. Sum over the possible combinations of
the \( m \) votes coming from insiders and outsiders.

\[
p_I(s, \nu, m) = \sum_{i=0}^{m} \left\{ \binom{s-1}{i} p_I^i (1-p_I)^{s-1-i} \left( \frac{N-s}{m-i} \right) p_O^{m-i}(1-p_O)^{N-s-(m-i)} \right\}
\]

\[
p_O(s, \nu, m) = \sum_{i=0}^{m} \left\{ \binom{s}{i} p_I^i (1-p_I)^{s-i} \left( \frac{N-s-1}{m-i} \right) p_O^{m-i}(1-p_O)^{N-s-1-(m-i)} \right\}
\]

The final beliefs \( q_j(s, \nu; k) \) are the sum over \( m \) such that the insider receives more votes than the outsider. In the event of a tie, a fair coin toss determines the outcome.

\[
q_j(s, \nu; k) = \sum_{m:m+\delta_k I > N-m-\delta_k I} p_I^m(s, \nu) + \frac{1}{2} \sum_{m:m+\delta_k I = N-m-\delta_k I} p_O^m(s, \nu)
\]  

(7)

**Definition 2.1.** A Markov perfect equilibrium in anonymous and symmetric strategies is

- Value functions \( V_I(s, \nu, \epsilon), V_O(s, \nu, \epsilon) \) as defined in (4)
- Policy functions \( \hat{\nu}_I(s), \hat{\nu}_O(s) \) as defined in (6) that solve the value functions
- Beliefs \( q_I(s, \nu; k), q_O(s, \nu; k) \) as defined in (7) that are consistent with the policy functions

### 2.4 Shareholder value

The expected value to shareholders of the board choosing the type \( k \) candidate in state \( (s, \nu) \) is

\[
\begin{align*}
V^I_S(s, \nu) &= w_S(s) + \beta \left[ \nu_I + \frac{s}{N} E[V_S(s, \nu')] + \left(1 - \frac{s}{N}\right) E[V_S(s+1, \nu')] \right] \\
V^O_S(s, \nu) &= w_S(s) + \beta \left[ \nu_O + \frac{s}{N} E[V_S(s-1, \nu')] + \left(1 - \frac{s}{N}\right) E[V_S(s, \nu')] \right]
\end{align*}
\]

where the expectation is unconditional over \( \nu' \). Given the policy functions of directors, the shareholder value function is

\[
V_S(s, \nu) = q(s, \nu)V_I^I(s, \nu) + (1-q(s, \nu))V_O^O(s, \nu)
\]  

(8)

where \( q(s, \nu) \) is the probability of an insider being voted in at state \( (s, \nu) \). Once a board has been established, shareholders play no strategic role in the model. However, when taking the firm public,
shareholders choose the initial composition of the board. For simplicity, I assume that shareholders do not get quality draws for the initial members of the board. The shareholders’ problem is

$$\max_{s_0 \in S} E[V_S(s_0, \nu)]$$  \hspace{1cm} (9)$$

where the expectation is unconditional over \(\nu\).

2.5 Regulation

Majority outsider regulation is easily implemented in the model. The regulation states that a board must be composed of a majority of outsiders. In the model, this puts a cap on \(s\). The set of states allowed under regulation is

$$S^{maj} = \{s : s \in \mathbb{Z}_+, s < N - s\}$$

Let \(V^{maj}_S(\cdot)\) be the shareholder value function under majority outsider regulation.

The nominating committee regulation is also easily implemented with the addition of the requirement that \(\sigma_i^2 = 0\). By taking randomness out of the individual shocks, all outside directors have the same preferences for candidates. With this assumption, all nominating committees composed entirely of outsiders are identical. The nominating committee requirement is then implemented in my model by having the outside directors’ preferences determine which candidate is chosen. Let \(V^{nom}_S(\cdot)\) be the shareholder value function under nominating committee regulation.

3 Analytical results

3.1 Static model

In the static version of the model with \(\beta = 0\), directors and shareholders only receive the work stage payoffs and no directors are hired or retire. The shareholder problem is simply to choose the board composition that maximizes the work stage payoff.

**Theorem 3.1.** In the static model, neither the majority outsider regulation nor the nominating committee regulation can increase shareholder value.
Proof. In the static model, the shareholder problem is

$$\max_{s \in S} w_S(s)$$

Because $S^{maj} \subset S$, majority regulation only reduces the set of compositions that shareholders choose from when setting up the board and can only hurt shareholder value. Because no new directors are hired in the static model, the nominating committee regulation does not change the shareholder problem.

It is clear that the majority policy will not benefit shareholders in the static model. Majority policy reduces the number of admissible compositions and will hurt shareholder value if the board that maximizes work stage payoffs is composed of a majority of insiders.

3.2 Director and shareholder comparison

From the work stage specification, it is clear that inside directors have preferences that differ from outsiders and shareholders. Shareholders and outside directors are not always in agreement either. First, outside directors have individual preference shocks over the candidates. Second, directors retire. In absence of carefully constructed retirement packages, the possibility of retirement changes a director’s preferences for the future. In the case of no retirement payments (as considered here), directors put less weight on the future than shareholders. To see these effects, fix outsider and shareholder continuation values at $V_S(s) = V_O(s) = V(s)$ with the i.i.d. shocks integrated out of the continuation values. At composition $s$, when will an outsider prefer an insider, but the shareholders prefer the outside candidate when

$$\nu_I - \nu_O > \epsilon_O - \epsilon_I + \frac{s}{N-1} [V(s-1) - V(s)] + \left(1 - \frac{s}{N-1}\right) [V(s) - V(s+1)]$$

and shareholders prefer the outside candidate when

$$\nu_I - \nu_O < \frac{s}{N} [V(s-1) - V(s)] + \left(1 - \frac{s}{N}\right) [V(s) - V(s+1)]$$
It is immediate that outsiders and shareholders differ in their distribution over future board compositions and outsiders have the individual $\epsilon$ preference shocks. The right hand side of the firm’s condition minus the right hand side of the outsider’s condition reveals that disagreement occurs when
\[
\frac{s}{N(N-1)}[2V(s) - V(s-1) - V(s+1)] + (\epsilon_I - \epsilon_O) \neq 0
\] (10)

Even with fixed continuation values, outside directors may want to take a one-shot deviation from the actions preferred by shareholders. Retirement issues may be resolved by paying incumbent directors a retirement package that eliminates the first term. Then in the absence of individual shocks, outside directors and shareholders are identical.

Suppose that outside directors and shareholders are identical. This would be the same as having shareholders choose replacement directors when the board is composed of a majority of outsiders. In this case the majority outsider regulation can only hurt shareholders by restricting the directors they are allowed to hire.

In the remaining analysis, I set the value of retirement to zero and fix the individual preference shocks at 0. Retirement is left as the only driver of shareholder and outsider divergence.

### 3.3 An analytically tractable case

In this section I present an analytically tractable version of the model that provides some intuition for the full model. In this version of the model, I shut down all randomness except for retirement. There are no individual shocks, and the quality draws are deterministic. I fix the outside candidate quality at 0 and let the inside candidate quality $\nu_I > 0$ vary. Additionally, I’m interested in the case where there is work stage conflict between inside and outside directors, so I let $B > \phi(s)$ for all $s$. I also take $\phi(\cdot)$ to be increasing in $s$, implying $w_O(0) = w_S(0) > w_O(1) = w_S(1)$. Last, for simplicity, I relax the requirement that the board contain one director of each type and allow
boards composed of all insiders or all outsiders.

\[ N = 3, \beta \in (0, 1) \]

\[ B > \phi(s) \quad \forall s \]

\[ \phi(s) \text{ strictly increasing in } s \quad (11) \]

\[ \mu_O = \sigma_O^2 = \sigma_I^2 = 0, \mu_I = \nu_I > 0 \]

\[ \mu_e = \sigma_e^2 = 0 \]

If the board is ever composed of a majority inside directors \((s \in \{2, 3\})\) it will stay that way forever. Insiders get work stage payoff of \(B\) in both states and \(\nu_I > 0\), so they will always hire the inside candidate. In these states shareholders and outside directors get work stage payoff 0, but they do receive the quality component of the new insider that are hired.

If the board has no insiders \((s = 0)\), both outsiders and shareholders would like to add an insider. First, the insider provides \(\nu_I > 0\) and second, \(w_O(1) > w_O(0)\). The question then, is what happens when there is already an insider on the board. If outsiders hire another insider, they get \(\nu_I > 0\), but risk sending the board to insider control where outsiders get work stage payoffs of 0 until retirement. If \(\nu_I\) is high enough, it is worth the risk of moving to insider control. Because directors retire and shareholders do not, the outside directors put less weight on the future and are more willing than shareholders to grab the \(\nu_I\) today and let the board move to insider control.

**Theorem 3.2.** Fix any parameters that satisfy (11). Then there exist cutoffs \(\hat{\nu}_O, \hat{\nu}_S\) such that

1. With \(s = 1\), outsiders hire the inside candidate iff \(\nu_I > \hat{\nu}_O\)

2. With \(s = 1\), the firm prefers the inside candidate iff \(\nu_I > \hat{\nu}_S\)

3. \(\hat{\nu}_O < \hat{\nu}_S\)

**Proof.** See appendix. \(\square\)

For the range \(\nu_I \in (\hat{\nu}_O, \hat{\nu}_S)\), outsiders and shareholders disagree about which candidate to hire. Since outsiders make the decision, there is room here for policy to improve shareholder value. I
return to this thought below.

If shareholders take a firm public with an initial board in state \( s_0 \), how will the composition evolve over time? Construct the \( 4 \times 4 \) transition matrix \( M \) where entry \( m_{ij} \) gives the probability of moving from state \( s = i \) to state \( s' = j \). Rows of \( M^t \) give the distribution of board compositions after \( t \) periods given that the board started in the state corresponding to the current row. Letting \( t \to \infty \) produces the invariant distribution.

Suppose \( \nu_I > \hat{\nu}_O \). Then both outside and inside directors will always vote for the inside candidate.

\[
M = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & \frac{1}{3} & \frac{2}{3} & 0 \\
0 & 0 & \frac{2}{3} & \frac{1}{3} \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad \lim_{t \to \infty} M^t = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

The first row of \( M \) shows that if the board starts with zero inside directors and an insider is hired, the board moves to one insider with probability one. In the second row, the board starts with one insider and another insider is hired. With probability \( \frac{1}{3} \) the incumbent insider retires and the board remains at one insider. With probability \( \frac{2}{3} \) an outsider retires and the board has two insiders in the next period. Notice that as time passes, no matter where shareholders start the board it will move to insider dominance. There is only one ergodic set, \( E = \{3\} \).

If \( \nu_I < \hat{\nu}_O \), then outsiders will stop voting for the inside candidates once there is already an insider on the board.

\[
M = \begin{pmatrix}
0 & 1 & 0 & 0 \\
\frac{1}{3} & \frac{2}{3} & 0 & 0 \\
0 & 0 & \frac{2}{3} & \frac{1}{3} \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad \lim_{t \to \infty} M^t = \begin{pmatrix}
\frac{1}{4} & \frac{3}{4} & 0 & 0 \\
\frac{1}{4} & \frac{3}{4} & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

Now shareholders can prevent the board from falling into insider control by starting the board with outsider control. Here there are two ergodic sets: \( E_1 = \{0, 1\} \) and \( E_2 = \{3\} \). Set \( E_1 \) is reached
when the board is started with outsider control and set $E_2$ is reached when the board is started in insider control.

Intuitively, if $\nu_I > \hat{\nu}_O$, shareholders know that the inside candidate will always be hired and the board will eventually move to insider dominance no matter the initial composition. Therefore shareholders want to start the board in outsider control in order to get work stage payoffs for as long as possible. If $\nu_I < \hat{\nu}_O$, shareholders still want to start with a majority outside board. In this case the board will stay in outsider control.

**Theorem 3.3.** Fix any parameters that satisfy (11). Let $s_0$ be the initial board state that maximizes shareholder value.

$$s_0 = \arg\max_{s \in S} V_S(s)$$

Then $s_0 \in \{0, 1\}$.

*Proof.* See appendix.

### 3.4 Majority outsider regulation

Majority regulation prevents the outside directors from voting in another insider when $s = 1$. Without any randomness (other than retirement), the majority policy does not alter behavior for small $\nu_I$. The outsiders will never vote in an insider when there is already one on the board and the policy never binds. On the other hand, if $\nu_I$ is very large and both shareholders and outsiders want to always hire an insider, policy forces an outsider and hurts firm value. The only time policy can help is when there is disagreement between outsiders and insiders: when $\hat{\nu}_O < \nu_I < \hat{\nu}_S$. In this case, policy prevents outsiders from hiring the extra insider and the shareholders benefit.

**Theorem 3.4.** Fix any parameters that satisfy (11). Suppose shareholders set up a new firm and are able to choose the initial board state.

1. If $\nu_I < \hat{\nu}_O$, majority policy does not change shareholder value.
2. If $\hat{\nu}_O < \nu_I < \hat{\nu}_S$, majority policy increases shareholder value.
3. If $\hat{\nu}_S < \nu_I$, majority policy decreases shareholder value.
Proof. See appendix.

Transition dynamics with majority policy are similar to the small $\nu_I$ case with no policy, except that the board may never be in insider control. For all $\nu_I > 0$,

$$M = \begin{pmatrix}
0 & 1 & 0 & 0 \\
\frac{1}{3} & \frac{2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad \lim_{t \to \infty} M^t = \begin{pmatrix}
\frac{1}{4} & \frac{3}{4} & 0 & 0 \\
\frac{1}{4} & \frac{3}{4} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

With regulation, the stationary distribution shows that the board will have zero insiders with probability $\frac{1}{4}$ and one insider with probability $\frac{3}{4}$. Whether or not this benefits shareholders depends on $\nu_I$.

4 Model with random quality

The analytical model provides good intuition, but it leaves out an important aspect of the model. Making director quality deterministic precludes any shocks that would make a firm temporarily prefer one candidate over another. If quality deterministically depended on the type of a candidate, it could be represented in the work stage and shareholders would like a policy that fixes the board composition at the composition that maximizes the firm’s work stage payoff. In reality, the board requires discretion to respond to short run needs of the firm and will receive candidates of differing quality. When the board’s decisions cannot be contracted on, giving directors discretion results in the possibility of the board making decisions that hurt shareholders. The need for flexibility is represented by increasing the variance of the distribution of quality draws. Adding this to the model comes at a cost: analytical solutions are no longer feasible. To study the more flexible model, I move to the computer and solve for numerical results.

In this section I follow recent trends in the industrial organization literature that use numerical techniques to analyze models that are not analytically tractable. See Judd (1997) for discussion of the validity and usefulness of numerical techniques in economic analysis. Doraszelski and Markovich
(2007) use techniques similar to mine in their study of dynamic advertising competition.

In this section, I set $\mu_I = \mu_O = 0$ and $\sigma^2_I = \sigma^2_O = \sigma^2$ and study how the variance of candidate quality affects firm value and director actions. In addition, I allow $B$ to vary in order to include cases where agency costs are low and there is no conflict between inside and outside directors.

4.1 Option value

Increasing $\sigma^2$ increases the importance of short run flexibility to the firm. But it is not always the case that the board can respond to these needs. Besides the quality draws, new directors affect value by changing the distribution of future board compositions though $E[V_j(s',\nu')|s,k]$. When choosing among candidates, there is an option value of having two quality draws, but it may be tempered by the compositional effects. In the extreme case where the compositional value does not change between candidate types, incumbents are free to hire the candidate with the maximum $\nu_k$. As the compositional value of one candidate becomes greater than the other, this option value is diminished.

This effect may be seen clearly by looking at the expected quality of the candidate actually chosen as a replacement. The exogenous $\nu$ draws are not affected by policy and are i.i.d. across time, so I integrate them out of the value function and analyze the value functions with the expected quality draw of the candidate eventually hired. This reveals the relationship between the compositional and option value aspects of the value functions. Let $\hat{\nu}(s)$ be the policy function for the type in majority on board $s$. The expected quality of the new director is

$$\frac{\sigma}{\sqrt{\pi}} \exp \left\{ -\frac{\hat{\nu}(s)^2}{4\sigma^2} \right\}$$

There are a few interesting observations to be made here. First, as $\sigma^2$ approaches 0, the option value goes to zero. If the board is drawing identical quality candidates, there is no benefit in having multiple draws. Second, the option value is decreasing as the cutoff function $\hat{\nu}(s)$ moves away from zero. Recall that this cutoff function gives the minimum advantage in quality an insider needs to have over the outsider for the board to choose the inside candidate. In other words, this is the compositional advantage of the outside candidate over the inside candidate. As this value moves
away from zero, the board favors one candidate over the other and diminishes the option value of drawing multiple candidates. The option value reaches its maximum when $\hat{\nu}(s) = 0$ and the board is free to pick the maximum quality candidate. When $\hat{\nu}(s) = \infty$, such as when regulation prevents the hiring of an insider, the option value is zero as the board must choose the outside candidate.

4.2 No policy

I now solve the model for some example firms in order to see the results discussed so far in action. The most interesting and illustrative case is $B$ large enough to cause work stage conflict between insiders and outsiders. Let $B > \phi(s), \forall s \in S$. An insider controlled board will always choose right in the work stage and an outsider controlled board will always choose left. Naturally an insider controlled board is very reluctant to let the board tip over to outside control and vice versa. Consider a firm with $N = 15$, $B = 2$, $\sigma^2 = 4$ and $\phi(\cdot) = 1 + 0.05s$. Director value functions and cutoff functions are shown in Figure 1. The value function for each type of director shows the expected present value of sitting on board $s$ before the candidates are drawn. The most dramatic feature occurs at the transition from outsider dominance to insider dominance. Board $s = \frac{N-1}{2} = 7$ is outsider controlled, but replacing an outsider with an insider would tip the board to insider control. If this happens, the value for an outsider decreases dramatically while the value for an insider rises sharply. Outsiders are only willing to vote for the inside candidate when the difference between candidates $\nu_I - \nu_O$ is high enough to offset the loss in majority from moving to an insider controlled board. This is revealed as a spike in the cutoff value $\hat{\nu}_O(s)$ at $s = 7$.

At boards heavily composed of one type of director, the board is in less danger of tipping over to the opposite type and the difference in compositional values of the candidates is not so dramatic. Consider a board with only one insider. The outside directors are in the majority and their preference determines the candidate hired. Hiring an insider still leaves the board solidly in outsider control and the continuation value of hiring an insider is very close to that of hiring an outsider. With only two insiders on the board, the outsiders slightly prefer hiring an insider over an outsider: $\hat{\nu}_O(2)$ is negative, meaning that outsiders will hire an insider that has a slightly lower quality draw than an outsider.
The importance of the option value of candidates is especially clear when looking at the value function of inside directors in an outsider controlled board. As the composition increases from no insiders to seven insiders, we might expect insider value to monotonically increase as the insiders get closer to taking control of the board. But the value function reveals that insider value begins decreasing as the board nears even composition. As equation 12 shows, the high cutoff value at \( s = 7 \) reduces option value. In this case the reduced option value outweighs the value to insiders of a higher probability of gaining majority control and the work stage benefit of an extra insider.

The shareholder value function is shown in Figure 2. The shareholder value function is very similar to the outsider value function in that shareholders experience a sharp drop in value when the board moves to insider control. The state that maximizes the shareholder value function is also the solution to the shareholders’ problem when taking a firm public. In this case, shareholders
would start the firm with three inside directors. This is in sharp contrast to the static version of this model in which the shareholders would choose a board with seven insiders. By starting the firm with fewer insiders, shareholders allow the board to respond to the randomly drawn quality component of candidates. Additionally, since the outsiders are not perfectly aligned with shareholders, there is room for outsiders to hire a few insiders that shareholders may not want without sending the board to a majority insider state.

Iterating on the $16 \times 16$ board state transition matrix $M$ reveals the distribution of board states as the board evolves through time. Row $j$ of $M^t$ specifies the distribution of board compositions after $t$ periods starting from an initial board with $j - 1$ insiders. Figure 4 shows the distribution after one, five and twenty periods. The invariant distribution is also shown and does not depend on the initial board – the only ergodic set is $S$. Because the quality draws have infinite support, there is always positive probability on a sequence of candidates that will lead to any admissible board composition. Convergence is slow though. Even after twenty periods, a board that begins with a majority of insiders is still controlled by insiders with very high probability.

4.3 Majority outsider regulation

![Graph showing shareholder value function with majority outsider regulation and no regulation](image)

Figure 3: Shareholder value function with majority outsider regulation and no regulation

Figure 3 shows the firm value function with the majority outsider requirement and with no policy. With policy, board compositions with $s > 7$ are not allowed so shareholder value is not
Figure 4: Transition matrix $M^t$ with no regulation

(a) $t = 1$

(b) $t = 5$

(c) $t = 20$

(d) $t \rightarrow \infty$

Figure 5: Transition matrix $M^t$ with majority outsider regulation

(a) $t = 1$

(b) $t = 5$

(c) $t = 20$

(d) $t \rightarrow \infty$
defined for these states. We see here that majority policy benefits shareholders at every admissible board composition.

Figure 5 shows the distribution of board compositions after one period, five periods, twenty periods and the invariant distribution. Here, there is zero probability that the board is composed of a majority of insiders.

Figure 6: Maximum shareholder value: Majority outsider regulation vs. No regulation

Majority policy clearly benefits the shareholders of our example firm, but is this always the case? Similar to the nominating committee analysis, I let \( V_{S}^{maj}(s) \) be the firm’s value function with the majority outsider policy. I vary \( \sigma^2 \) and \( B \) over a fine grid to see where \( \max_{s \in S} V_{S}^{maj}(s) \geq \max_{s \in S} V_{S}(s) \). Results are shown in Figure 6.

If \( B \) is small enough that a board with a majority of insiders is optimal, then the majority outsider policy will clearly hurt the firm. When \( B \) is large, there is an endogenous barrier that dissuades outsiders from hiring enough insiders that the board tips to insider control. When \( \sigma^2 \) is small, it is very unlikely to draw candidates such that outsiders are willing to move to insider
control. In this case, majority policy only changes behavior with low probability. When $\sigma^2$ is higher, it is more likely that outsiders will tip the board to insider control and the policy has more bite. An interesting feature of the diagram is that as $\sigma^2$ increases, the minimum $B$ for policy to benefit shareholders is also increasing. The combination of high variance of candidate quality and low $B$ (but still high enough to cause work stage conflict) mean that it is more likely that an insider controlled board will return to outsider majority. When $B$ is low, insiders do not give up much by losing control and the high variance increases the probability of getting a high quality outsider. In these cases, shareholders prefer no majority regulation so that the board can remain flexible and capture the high option value that comes with a high variance. As $\sigma^2$ increases the benefits of being flexible increase and offset the negative effects of insider control with a higher $B$.

4.4 Alternative regulations

In 2003 the NYSE and NASDAQ exchanges also implemented a nominating committee regulation. This new rule states that replacement directors must be selected by a committee composed of outside directors. The optimal nominating committee would be shareholders themselves, but as that is not possible, outside directors are the second best. In my model this rule is implied by the majority outsider regulation, but it is also interesting to see how this regulation would affect shareholders in the absence of the majority regulation. I implement the nominating committee requirement by letting outsider preferences determine the hiring decision. Let the firm's value function under the nominating committee requirement be denoted by $V_{S}^{nom}(s)$.

Figure 7 shows the firm value function of the example firm with a nominating committee and without. Without policy, moving from seven to eight insiders causes a large drop in firm and outsider value. The nominating committee reduces the penalty of letting the board go to a majority of inside directors as outsiders remain in control of the hiring decisions and are more likely than insiders to hire directors that return the board to outside majority. This reduces $\hat{\nu}_O(7)$, which in turn increases the option value of quality draws at $s = 7$.

Figure 8 shows the distribution of board compositions after one period, five periods, twenty periods and the invariant distribution. Compared to the firm without regulation, the nominating
committee keeps the board in outsider control with higher probability. If the board is started with a majority of insiders, it is very likely that a board with a nominating committee has moved to a board with a majority of outsiders after twenty periods. Without a nominating committee, the board is more likely to be in insider control than outsider control.

In our example firm, nominating committees increase firm value at all compositions and shareholders would rather take a firm public in the nominating committee environment rather than the no policy environment. Is this always the case? Let $V_{nom}^S(s)$ be the firm’s value function with the nominating committee policy. I vary $\sigma^2 \in [0, 11]$ and $B \in [1, 9]$ over a fine grid to see where $\max_{s \in S} V_{nom}^S(s) \geq \max_{s \in S} V_S(s)$. This experiment has been performed with various board sizes and discount factors and results are similar. Figure 9 shows the results.

Over most of the parameter space the nominating committee policy benefits shareholders. This is intuitive as the policy does not limit permissible board compositions and outsiders are more closely aligned with shareholders than insiders and outsiders retain control over hiring decisions in all states. A nominating committee composed of shareholders would always be better than no regulation, but that is not the case when the nominating committee is composed of outside directors. There is a range of the parameter space where shareholders prefer the no regulation environment. When $\sigma^2$ is small, two things happen. First, since the variance of the quality draws
is low, it is unlikely that the board receives candidates that make the outsiders want to take the
to insider control and the hiring control benefits of the nominating committee are rarely
realized. Second, we have seen that benefit of the nominating committee is that it increases
the option value of drawing two candidates when the board is near even composure. When $\sigma^2$
is small, this option value is small to begin with and there is not much to be gained with the
nominating committee. The magnitude of the difference in shareholder value with and without
policy is explored below.

4.5 Regulation Comparison

How do the two regulations compare to each other? Intuitively, the nominating committee policy
preserves flexibility in composition while helping to align incentives in hiring decisions. The majority
policy also aligns incentives, but does so at the cost of limiting admissible compositions.

Figure 10 compares nominating committees and majority policy in the example firm. In all
states, shareholders prefer the nominating committee policy to the majority policy and no policy.
It is not always the case that the nominating committee regulation alone is better than the majority outsider policy. I find that when the nominating committee policy is not optimal, implementing it will not hurt shareholders very much. When the nominating committee is optimal, it has the potential to greatly increase shareholder value. To explore the magnitude of regulation’s affects on shareholder value, I fix $B$ and plot the maximum firm value as $\sigma^2$ varies. Two examples are shown below.

When $B$ is low (Figure 11), agency costs are low and there is no tension between insiders and outsiders. Majority policy prevents the board from choosing a composition with a majority of insiders and shareholders suffer. The difference between no policy and nominating committees is small. When $B$ is large (Figure 12), we see that the nominating committee almost always outperforms majority policy. In the small range of $\sigma^2$ where the majority policy is better, the difference is small. Not much is lost in this case. On the other hand, as $\sigma^2$ increases, nominating committees have the potential to give shareholders large gains over no policy and majority policy. For both $B$, we see that nominating committees are preferable to no policy and majority policy.
5 Conclusion

The notion that directors choose their own board is of great importance when modeling the board and performing policy analysis. It introduces a dynamic aspect to the director selection problem and allows the endogenous determination of composition. By including the task of hiring new directors to the list of board functions, I am able to model entrenchment and other board dynamics. This is especially important for analyzing policies that target board composition and director hiring decisions.

This type of model is easily adapted to analyze other dynamic aspects of the board. Bebchuk et al. (2002) examine how staggered boards reduce the probability of a hostile takeover by increasing the time necessary for a hostile bidder to gain a majority of seats on a board. My model may be extended to include director term lengths and used to study staggered and annually elected boards and their effects on shareholder value.
Figure 11: $B = 1$

Figure 12: $B = 5$
The idea of incumbent committee members determining the future composition of the committee is not unique to corporate boards. For example, an economics department hiring a new faculty member considers both how the candidate will contribute to the output of the department and how the new faculty member will vote in future hiring rounds. A department may receive a very high quality macroeconomist, but hiring the candidate would result in giving too much weight to the macroeconomists in future hiring decisions. The ideas in my model may be adapted to these other environments.

There is also interesting work to be done in studying the response of firms to the policies discussed in this paper. Analysis here is limited to a new firm entering one of three policy regimes. Transition dynamics of incumbent firms is another avenue of exploration. The majority policy is seen to reduce the option value of picking among candidates. Firms affected by this could regain option value by increasing the size of their board. When there is uncertainty over how directors will vote in hiring decisions, the majority outsider policy may also act as a commitment mechanism. In this case, firms may actually increase the number of insiders on their board and they are able to get closer to the statically optimal level of insiders without the the possibility of the board tipping to insider majority.
References


A Proofs

A.1 Proof of Theorem 3.2

By assumption, if insiders take control of the board \((s \in \{2, 3\})\), then insiders always vote in the inside candidate and the board never returns to outsider control. We have

\[
V_I^O(0) = w_S(0) + \frac{2}{3} \beta [\nu + V_O(1)] \\
V_O^O(0) = w_S(0) + \frac{2}{3} \beta [V_O(0)] \\
V_O(0) = \max\{V_I^O(0), V_O^O(0)\} \\
V_I^O(1) = w_S(1) + \frac{2}{3} \beta \left[ \nu + \frac{1}{2} V_O(1) + \frac{1}{2} V_O(2) \right] \\
V_O^O(1) = w_S(1) + \frac{2}{3} \beta \left[ \frac{1}{2} V_O(0) + \frac{1}{2} V_O(1) \right] \\
V_O(1) = \max\{V_I^O(1), V_O^O(1)\} \\
V_O(2) = V_O(3) = \frac{\nu}{1 - \frac{2}{3} \beta} \\
V_S(2) = V_S(3) = \frac{\nu}{1 - \beta}
\]

Suppose parameters are such that outsiders prefer to hire an insider with \(s = 1\). Then

\[
\nu + \frac{1}{2} V_O(1) + \frac{1}{2} V_O(2) > \frac{1}{2} V_O(0) + \frac{1}{2} V_O(1) \\
\nu > \frac{1}{2} \left[ V_O(0) - \frac{\nu}{1 - \frac{2}{3} \beta} \right]
\]

Additionally, \(V_O(1) = V_O^I(1)\) and is now one equation with one unknown.

\[
V_O^I(1) = w_S(1) + \frac{2}{3} \beta \left[ \nu + \frac{1}{2} V_O^I(1) + \frac{1}{2} V_O(2) \right] \\
V_O^I(1) = \frac{\beta [(9 - 4\beta) \nu - 6w_O(1)] + 9w_O(1)}{9 + \beta (2\beta - 9)}
\]

If the outsiders vote in an insider with \(s = 1\), what do they do when \(s = 0\)? They vote for the
insider if

$$\nu > V_O(0) - V_O(1)$$
$$\nu > V_O(0) - \frac{\beta[(9 - 4\beta)\nu - 6w_O(1)] + 9w_O(1)}{9 + \beta(2\beta - 9)}$$
$$\nu > \frac{V_O(0)}{1 + \frac{\beta(9 - 4\beta)}{9 + \beta(2\beta - 9)}} - \frac{3w_O(1)(2\beta - 3)}{2\beta^2 - 9}$$

This is a less restrictive condition than the $s = 1$ condition, meaning that outsider vote for the insider with $s = 1$ then they also do so for $s = 0$. This is intuitive. It also tells us that in this case $V_O(0) = V_O^I(0)$ and is now solved for.

$$V_O(0) = w_O(0) + \frac{2}{3}\beta(\nu + V_O(1))$$
$$= w_O(0) + \frac{2}{3}\beta\left(\nu + \frac{\beta[(9 - 4\beta)\nu - 6w_O(1)] + 9w_O(1)}{9 + \beta(2\beta - 9)}\right)$$

So we have all of the outsider value functions solved for. By looking where the outsiders are indifferent between hiring an insider and an outsider with $s = 1$, we get the cutoff $\hat{\nu}_O$.

$$\hat{\nu}_O = \frac{1}{2} \left[ V_O(0) - \frac{\nu}{1 - \frac{2}{3}\beta} \right]$$
$$\hat{\nu}_O = \frac{6\beta w_O(1) + 3(3 - \beta)w_O(0)}{27 - \beta(9 + 2\beta)}$$

Because the outside director determines the hiring outcome when $s = 0, 1$, we know that if the outsider want an insider when $s = 1$, then $V_S(0) = V_S^I(0)$ and $V_S(1) = V_S^I(1)$. Then, similar to the procedure for the outsiders, we can calculate $\hat{\nu}_S$ and get

$$\hat{\nu}_S = \frac{3(\beta + 1)w_S(1) - (\beta - 3)w_S(0)}{(5 + \beta)(3 - 2\beta)}$$

Now, check to see that $\hat{\nu}_S - \hat{\nu}_O > 0$. Since $w_S(0) = w_O(0)$ and $w_S(1) = w_O(1)$, I replace the work stage functions with $w(0)$ and $w(1)$.
\[
\frac{3(\beta + 1)w_S(1) - (\beta - 3)w_S(0)}{(5 + \beta)(3 - 2\beta)} - \frac{6\beta w_O(1) + 3(3 - \beta)w_O(0)}{27 - \beta(9 + 2\beta)}
\]

= \[
\frac{w(0)(54\beta - 4\beta^3 - 54) + 3w(1)(27 + \beta(3 + 2\beta)) - 12)}{(5 + \beta)(3 - 2\beta)(27 - \beta(9 + 2\beta))}
\]

The denominator is positive. The first term with \(w(0)\) is negative and the term with \(w(1)\) is positive. By assumption, \(w(0) \leq w(1)\), so if the nominator is positive with \(w(1) = w(0)\), then the result holds.

\[
\frac{w(0)(54\beta - 4\beta^3 - 54) + 3w(1)(27 + \beta(3 + 2\beta)) - 12)}{(5 + \beta)(3 - 2\beta)(27 - \beta(9 + 2\beta))}
\]

= \[
\frac{w(0)(3 + \beta)(9 + \beta(3 + 2\beta))}{(5 + \beta)(3 - 2\beta)(27 - \beta(9 + 2\beta))}
\]

### A.2 Proof of Theorem 3.3

Case 1: Suppose \(\nu > \hat{\nu}_O\) and all directors will always vote in the inside candidate. Then shareholders get \(\nu\) in every period regardless of where the board starts. If the board starts in insider control, then shareholders never get any work stage payoff. If the board starts in outsider control then the board gets a positive work stage payoff in at least one period. Therefore \(s_0 \in \{0, 1\}\).

Case 2: Suppose \(\nu < \hat{\nu}_O\) and outside directors will hire the outside candidate when \(s = 1\). If \(\nu < \hat{\nu}_O\), then it must be that \(\nu < \hat{\nu}_s\). For shareholders to prefer the outside candidate when \(s = 1\), it must be the case that

\[
\nu + \frac{1}{3}V_S(1) + \frac{2}{3}V_S(2) < \frac{1}{3}V_S(0) + \frac{2}{3}V_S(1)
\]

\[
\nu < \frac{1}{3}(V_S(0) + V_S(1) - 2V_S(2))
\]

Since \(\nu\) is positive, it must be that either \(V_S(0) > V_S(2)\) or \(V_S(1) > V_S(2)\) or both and the result follows.
A.3 Proof of Theorem 3.4

Outline of proof...

Case 1: If $\nu < \hat{\nu}_O$, then the board starts in outsider control and outsiders hire the outside director when $s = 1$ even without regulation. In this case, regulation never changes agent behavior and values remain constant.

Case 2: If $\nu < \hat{\nu}_O$, outside directors would like to vote in an inside director when $s = 1$ but shareholders prefer the outsider. Then $V_S^I(1) < V_S^O(1)$, but $V_S(1) = V_S^I(1)$. With regulation, $V_S(1) = V_S^O(1)$ and shareholders are better off.

Case 3: If $\nu > \hat{\nu}_S$, both shareholders and directors would like to vote in the inside candidate when $s = 1$. Regulation prevents this, and all agents are worse off.