Linear algebra, Spring 2017 — Eigenstuff

Class on April 6

Exercise 1. Suppose A is a $n \times n$ matrix with eigenvector $\vec{v}$ and corresponding eigenvalue λ. That is,

$$A\vec{v} = \lambda\vec{v}.$$

1. Is $\vec{v}$ an eigenvector of $A + 2I_n$? If so, what is the eigenvalue?
2. If $\vec{v}$ is also an eigenvector of another $n \times n$ matrix B, is it necessarily an eigenvector of $A + B$?
3. What can you say about the kernel of $A - \lambda I_n$? Is $A - \lambda I_n$ invertible?

Exercise 2. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

1. Compute $A\vec{x}$.
2. Use your previous answer to compute $A^2\vec{x}$.
3. Use your previous answer to compute $A^3\vec{x}$ and after that compute $A^4\vec{x}$.
4. Do you see a familiar sequence starting to form? What would $A^{100}\vec{x}$ be? If you're patient, you can compute this by hand, but is there another approach?