Math 211, Spring 2017 — Homework 9

Tim Chumley

Due April 20

Problem 1 (Exercieses 7.1.4, 6)**.** Let A and B be $n \times n$ matrices, and $\vec{v}$ an eigenvector of A with associated eigenvalue λ and an eigenvector of B with associated eigenvalue μ.

1. Is $\vec{v}$ an eigenvector of $7A$? If so, what is the eigenvalue?
2. Is $\vec{v}$ an eigenvector of AB? If so, what is the eigenvalue?

Problem 2 (Exercises 7.1.16, 18, 20, 21)**.** Arguing geometrically, find all eigenvectors and eigenvalues of the following linear transformations.

1. Rotation through an angle of 180 degrees in $\mathbb{R}^2$.
2. Reflection about a plane V in $\mathbb{R}^3$.
3. Rotation about the $\vec{e}_3$-axis through an angle of 90 degrees, counter-clockwise as viewed from the positive $\vec{e}_3$-axis in $\mathbb{R}^3$.
4. Scaling by 5 in $\mathbb{R}^3$.

Problem 3 (Exercise 7.1.38)**.** We are told that $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$; what is the associated eigenvalue?

Problem 4 (Exercises 7.2.2, 4, 6, 8, 10)**.** For each of the following matrices, find all the real eigenvalues and their algebraic multiplicities.

1. $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 2 & 1 & 2 & 1 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 4 \\ -1 & 4 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
4. $\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$
5. $\begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix}$

Problem 5 (Exercises 7.3.6, 8, 10, 12, 14)**.** For each of the following matrices, find all real eigenvalues. Then find a basis of each corresponding eigenspace.

1. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ -5 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$