Linear algebra, Spring 2017 — Matrix Inverse

Class on February 17

Exercise 1. Let $a, b \in \mathbb{R}^2$ be numbers such that $a^2 + b^2 = 1$. Find the inverse of the following matrix using the 2×2 matrix inverse formula:

$$A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}.$$

By interpreting A as a geometric linear transformation, explain why the A^{-1} you found was to be expected.

Exercise 2. Let

$$A = \begin{bmatrix} 1-k & 2 \\ 4 & 3-k \end{bmatrix}.$$

For what values of k is A invertible?

Exercise 3. Let

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

1. What geometric actions are performed by A and R?
2. We claim AR is invertible. Find $(AR)^{-1}$ without using the 2×2 matrix formula; instead, think geometrically and use A^{-1} and R^{-1}.
3. What is $(RA)^{-1}$?