Math 211, Spring 2017 — Quiz 4

March 9

Name: ____________________

Instructions. Please do all of the following problems. No calculators or other materials are allowed.

Problem 1. Complete the following definitions.

Let $T : \mathbb{R}^m \to \mathbb{R}^n$ be a linear transformation. The **image** of T is...

any of the following is acceptable

$$\left\{ T(\vec{x}) : \vec{x} \in \mathbb{R}^m \right\} = \left\{ \vec{b} \in \mathbb{R}^n : T(\vec{x}) = \vec{b} \text{ for some } \vec{x} \right\}$$

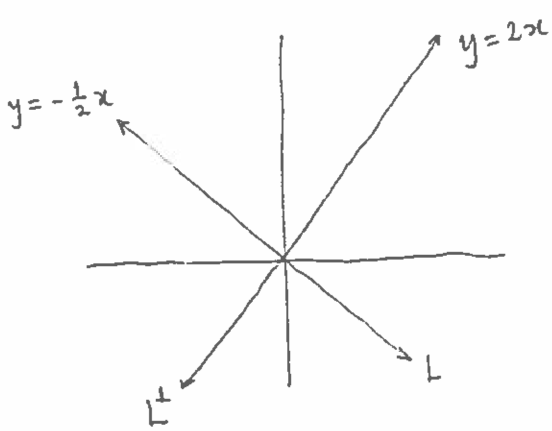
$=$ "the values T takes in the target space $\mathbb{R}^n$"

The **kernel** of T is ...

$$\left\{ \vec{x} : T(\vec{x}) = \vec{0} \right\} = \text{"solution set of } T(\vec{x}) = \vec{0}\text{"}$$

$=$ "the zeroes of T"

Problem 2. Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\ker T = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = 2x \right\}$.



Consider the line L whose graph $y = -\frac{1}{2}x$ is perpendicular to $y = 2x$.

Let $T(\vec{x}) = \text{proj}_L(\vec{x})$.

Then T has the desired kernel since everything on $y = 2x$ line maps to $\vec{0}$.

Problem 3. Determine whether the set $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x^2 + y^2 + z^2 = 1 \right\}$ is a subspace of $\mathbb{R}^3$. If it is, show that the three defining properties of a subspace hold. If it isn't, show that one of them fails to hold.

No a subspace

- The property of closure under scalar multiplication fails to hold. Every vector $\vec{x} \in S$ must have length $\|\vec{x}\| = 1$. So for any $k \neq 1$,

$$\|k\vec{x}\| = |k|\,\|\vec{x}\| = |k| \neq 1.$$

So $\|kx\| \neq 1$ and $k\vec{x} \notin S$.

- Also $\vec{0} \notin S$, and not closed under addition either.

Problem 4. Consider the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \end{bmatrix}$. Describe the image of A geometrically, ie. as a line, plane, or something else by first writing the image as a span of as few vectors as possible.

Note. $\operatorname{im}(A) = \operatorname{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$

which means it's a line. (in the direction of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$).