Math 211, Spring 2017 — Quiz 6

March 31

Name: ______________________________

Instructions. Please do all of the following problems. No calculators or other materials are allowed.

Problem 1. Consider the basis $\mathcal{B} = (\vec{v}_1, \vec{v}_2)$ where

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

1. If $\vec{x} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ then find the $\mathcal{B}$-coordinate vector $[\vec{x}]_{\mathcal{B}}$.

Notice $\vec{x} = \vec{v}_1 + 2\vec{v}_2$

So $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

2. If $[\vec{y}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ then find the vector $\vec{y}$ in standard coordinates.

$$\vec{y} = 3\vec{v}_1 + 3\vec{v}_2$$

$$= 3\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 3\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 12 \end{bmatrix}$$

Problem 2. Consider the plane V in $\mathbb{R}^3$ given by the equation

$$2x_1 + 3x_2 - x_3 = 0$$

and let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation which reflects vectors across V.

1. Find a vector $\vec{v}_1$ perpendicular to the plane.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

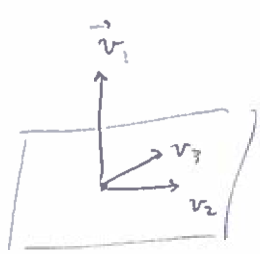
2. Find two vectors $\vec{v}_2, \vec{v}_3$ which are on V but not parallel to each other.

Let $x_1 = 1,\ x_2 = 0$. Then $x_3 = 2$

$x_1 = 0,\ x_2 = 1$. Then $x_3 = 3$

So $\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ are in V.

3. Let $\mathcal{B} = (\vec{v}_1, \vec{v}_2, \vec{v}_3)$. Find the $\mathcal{B}$-matrix B of T.



$T(\vec{v}_1) = -\vec{v}_1$

$T(\vec{v}_2) = \vec{v}_2$ So $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$T(\vec{v}_3) = \vec{v}_3$

4. Express the standard matrix A of T as a product of 3 matrices.

$$\begin{tikzcd} x \arrow[d, "S^{-1}"'] & T(\vec{x}) \\ {[x]_{\mathcal{B}}} \arrow[r, "B"'] & {[T(x)]_{\mathcal{B}}} \arrow[u, "S"'] \end{tikzcd}$$

$A = SBS^{-1}$

where $S = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix}$