

Math 211, Spring 2017 — Quiz 7

April 7

Name: _____

Instructions. Please do the following problems. No calculators or other materials are allowed.

Problem 1. Compute the determinant of

$$A = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 3 & 5 & 0 & -2 \\ 1 & 1 & 0 & -3 \\ 4 & 0 & 3 & -1 \end{bmatrix}.$$

Expanding across 3rd column gives

$$\det A = \det \begin{bmatrix} 3 & 5 & -2 \\ 1 & 1 & -3 \\ 4 & 0 & -1 \end{bmatrix} - 0 + 0 - 3 \det \begin{bmatrix} 2 & -1 & 0 \\ 3 & 5 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

$$= 4 \det \begin{bmatrix} 5 & -2 \\ 1 & -3 \end{bmatrix} - \det \begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix} - 3 \left(2 \det \begin{bmatrix} 5 & -2 \\ 1 & -3 \end{bmatrix} + \det \begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix} \right)$$

$$= -2(-15+2) - (3-5) - 3(-9+2)$$

$$= 26 + 2 + 21$$

$$= 49.$$

Problem 2. Answer 2 of the following 3 true/false questions. Please make sure to provide a short justification and circle which you'd like to be graded.

1. $\det(A^5) = (\det A)^5$ for all 5×5 matrices A .

True

$$\begin{aligned}\det(A^5) &= \det(\underbrace{A \cdots A}_{5\text{-fold product}}) \\ &= \underbrace{\det(A) \cdots \det(A)}_{5\text{-fold product}} \quad \left(\begin{array}{l} \text{using } \det(AB) \\ = \det A \det B \end{array} \right) \\ &= (\det A)^5\end{aligned}$$

2. If all the entries of a 7×7 matrix A are 7, then $\det A$ must be 7^7 .

False $\det(A) = 0$ since it has equal rows.

3. The equation $\det(-A) = \det A$ holds for all 6×6 matrices.

True

$$\begin{aligned}\det(-A) &= (-1)^6 \det A \quad \left(\begin{array}{l} \text{in general, } \det(kA) \\ = k^6 \det A. \end{array} \right) \\ &= \det(A)\end{aligned}$$