# Linear algebra, Spring 2017 — Rank of a matrix

Class on February 2

Suppose we have a linear system with $n$ equations and $m$ unknowns. We've learned that this system can be expressed as the equation $A\vec{x} = \vec{b}$ or as an augmented matrix

$$\left[\begin{array}{c|c} A & \vec{b} \end{array}\right]$$

where $A$ is an $n \times m$ matrix called the coefficient matrix and $\vec{b} \in \mathbb{R}^n$.

**Exercise 1.** Suppose we are told that the linear system is inconsistent. What important feature should $\text{rref}(A)$ have? Can you give an example of what $\text{rref}(A)$ might look like if $A$ is a $5 \times 3$ matrix? How many leading 1's can your example have? What can you say about $\text{rank}(A)$?

**Exercise 2.** Suppose we are told that the linear system has infinitely many solutions. What should $\text{rref}(A)$ look like this time? Can you give an example of what $\text{rref}(A)$ might look like if $A$ is a $5 \times 3$ matrix? How many leading 1's can your example have? What can you say about $\text{rank}(A)$?

**Exercise 3.** Suppose we are told that the linear system has exactly one solution. What should $\text{rref}(A)$ look like this time?

1. Can you give an example of what $\text{rref}(A)$ might look like if $A$ is a $5 \times 3$ matrix? How many leading 1's can your example have? What can you say about $\text{rank}(A)$?

2. Could $A$ be a $3 \times 5$ matrix? Why or why not?