Measurement of the acceleration due to gravity

Introduction:

An object which is released near the surface of the earth will accelerate under the influence of gravity. This motion is referred to as free fall. If the object is fairly dense, we can ignore any effect on its motion due to air resistance and we can consider the acceleration \( a \) to be a constant so that \( a = g \).

The kinematic equations of motion for constant acceleration (below) can be used to describe the position \( y \) and the speed \( v \) of a body in free fall as a function of time \( t \).

\[
\begin{align*}
(1) \quad y &= y_i + v_i t + \frac{1}{2} g t^2 \\
(2) \quad v &= v_i + g t 
\end{align*}
\]

The apparatus consists of an electromechanical device which will release an object into free fall. As it falls, a sparker mechanism produces a spot on a strip of red waxed paper every 1/30 of a second, providing a record of the body’s position at evenly spaced time intervals. From this data, the value of \( g \) can be determined.

Procedure:

Your lab instructor will assist you in making a spark tape. The release mechanism is not synchronized with the sparker, so the \( v=0 \) position is unknown. Also, residual magnetism when the mass starts to fall causes the points nearest the top to be invalid.

Ignoring the first few points, label the spots on the tape \( y_o, y_1, y_2, \) etc. as indicated in the figure. Use a 2-meter stick to make a careful measurement of the distance from \( y_o \) to each other spot. Choose \( y_o=0 \) for a convenient origin. Estimate the uncertainty in your length measurements and include this with your data. At this point, you are ready to find \( g \).

But how? One can imagine analyzing the data numerically. For example, the separation between pairs of adjacent spots could be divided by the time interval (call it \( T \)) between spots to give a series of values for velocity \( v \) as a function of increasing time. Then pairs of \( v \) values could similarly be divided by \( T \) to produce a number of values for \( g \) which could then be averaged together. It turns out that this method would not be very efficient in making use of your full set of data. Consider the following. Suppose you had seven data points labeled \( y_o, y_1, \ldots, y_6 \) and decided to combine them in pairs to get 6 values of average velocity, \( v_o, v_1, \ldots, v_5 \), as shown schematically below. Similarly, you could calculate 5 values of average acceleration \( a_o, a_1, a_2, a_3, a_4 \) as shown.
If you averaged all of these values of \( a \) together to get an overall value for \( g \), the result would be:

\[
g = \frac{a_0 + a_1 + a_2 + a_3 + a_4}{5} = \frac{v_5 - v_0}{5T} = \frac{y_6 - y_6 - y_6 + y_6}{5T^2}
\]

What a waste of data! This method would only use the top two and bottom two points, ignoring all of the information from the others. This effect would be even more dramatic in your case if you have more than 7 points, which is certainly the case in this lab. Clearly this is not the method we will use. Instead, we’ll use a graphical approach.

**Analysis:**

1) Use KaleidaGraph or Excel to make a graph of position \( y \) vs. time \( t \) for the falling object. You can set your time origin so that the first good point \( y_0 \) occurs at \( t=0 \). Do you recognize the shape as a curve for constant acceleration? Perform a second order polynomial fit to the data. Using the equation of the best fit line, determine your value for \( g \).

2) Make a second plot, this time for velocity vs. time, using the following method. Essentially, you’d like to take the derivative of the position plot. You could find a series of values of average velocity by pairing up neighboring points (as in the previous discussion). However, to what values of \( t \) would these \( v \)’s correspond? Instead, make pairs with every second data point. For the example of only 7 points, using the diagram below as a guide,
Here, $v_o$ is the average velocity between $y_2$ and $y_o$. But it is also (conveniently) the instantaneous velocity at $y_1$ (where $t = T$). Similarly, $v_1$ is the average velocity between $y_3$ and $y_1$ and also the instantaneous velocity at $y_2$ (where $t = 2T$), etc.

Convince yourself that this is true; i.e. show that

$$\bar{v} = \frac{y_3 - y_1}{2T} = v_2$$

where you know that $v_2 = v_1 + gt_2 = v_i + g2T$ from Eq. 2 on the first page.

You can either calculate all of the velocity values by hand, or use the formula function of Excel or KaleidaGraph to calculate the velocity values for you. Using the calculated velocity values, make a velocity plot the old-fashioned way, by hand, using a piece of graph paper. Look at the velocity plot. Is the shape of the curve consistent with what you would expect for constant acceleration? Draw a best-fit line through your data. What is its slope? Is it what you expect? Use this information to determine your second value for $g$.

3) Make a third plot, an acceleration vs. time plot, in a manner similar to the way the velocity plot was made. That is, calculate the values of average acceleration by using every second value of $v$ (as shown schematically in the diagram above), and plot (by hand) these values of $a$ vs. $t$ so that $a_o$ is the acceleration corresponding to the point $y_2$ (where $t = 2T$), etc. What do you expect for the slope of the straight line through this data? Draw a best-fit horizontal line through your data. This is essentially the graphical equivalent to taking a numerical average of all your $a$ values. Where does it intercept the acceleration axis? Use this information to determine a third value for $g$?

4) Take a numerical average of all your acceleration values, and compare the result to the $g$ value that came from the acceleration plot. They should be very similar. Average the three values of $g$ you obtained from each of the plots. Compare your results to the handbook value of $g$ in South Hadley (980.4cm/s$^2$) and determine the percent error in your measured results.

Questions:

To what can you attribute any uncertainties in your calculated values for $g$ as far as the experimental techniques and equipment are concerned? How much error do you think these would impose on your results?