Purpose

To investigate the motion of a launched ball, in two dimensions, and study the relationship of range and launch angle.

Theory

A ball leaving a launcher, at an angle $\theta$ from the horizontal, with velocity $v_0$, undergoes motion in two dimensions, $x$ and $y$. These motions are completely independent of each other: the vertical ($y$) component of the initial velocity $v_0$ undergoes a constant acceleration (downwards, at -9.81 m/s²), while the horizontal ($x$) component of the initial velocity $v_0$ remains unchanged. In other words, the vertical component of motion is governed by the following equations:

1) \[ y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]

2) \[ v_y^2 = v_{0y}^2 - 2gy_0 \]

3) \[ v_y = v_{0y} - gt \]

while the horizontal component of motion is governed by:

4) \[ x = x_0 + v_x t \]

where $v_{ox} = v_x = v_0 \cos(\theta)$, $v_{oy} = v_0 \sin(\theta)$. This type of motion (constant acceleration in one dimension, constant velocity in another) is called projectile motion.

Consider a ball launched from the surface of a table, at angle $\theta$, with velocity $v_0$. How far will it have traveled on the table before it hits the table again?

To answer this, we need to determine how long the ball is in the air. Consider equation 4, the initial x position is zero, so the time it takes to get to its final horizontal position, $R$, is:

5) \[ t = \frac{R}{v_x} \]
If we define the range as \( R = x - x_0 \) and set \( y = y_0 \) (since the ball will be at its initial height when it lands, we can write equation 1 as

\[
6) \quad \theta = \frac{v_0}{v_x} \cdot \frac{1}{2} g \left( \frac{R}{v_x} \right)^2 = \frac{v_0 \sin(\theta) R}{v_0 \cos(\theta)} - \frac{1}{2} g \left( \frac{R}{v_0 \cos(\theta)} \right)^2 = R \tan(\theta) - \frac{g R^2}{2 v_0^2 \cos^2(\theta)}
\]

If we solve the last using the quadratic equation for \( R \), (ignoring the solution when \( R = 0 \)) we have

\[
7) \quad R = \frac{v_0^2}{g} \sin(\theta) \cos(\theta) = \frac{v_0^2}{g} \sin(2\theta)
\]

Thus, the range of a projectile (our launched ball) just depends on the initial velocity and the launch angle. And, perhaps surprisingly, the range of projectiles at \( \theta = 30^\circ \) and \( \theta = 60^\circ \), with the same initial velocity, is the same! We will test this assertion, as well as confirm equation 7.

**Procedure**

Your launcher can be set to launch the ball at any angle from 0 to \( 90^\circ \). This angle is measured by checking the position of the plumb bob. Be sure to always launch your ball from the same position (same number of clicks). Also be sure to wear goggles if you are in the direction of the launch path. Warn people before launching by saying “ready, aim, FIRE!” or words to that effect.

**Part I: Initial Velocity**

Make sure that your launcher is firmly clamped to your lab table, and angle the launcher so that it aims upwards \( (\theta = 90^\circ) \). With a 2 meter stick next to the launcher, launch the ball vertically \( (\theta = 90^\circ) \), and measure the height \( h \) that the ball reaches. Do this by adjusting the position of a sliding pipe cleaner attached to the stick until the pipe cleaner marks the height of your shots. Repeat this five times, and then take the average of these heights. Note that the height of the table is the same as the launch point, so use that as a reference height \( (h = 0) \).

<table>
<thead>
<tr>
<th>Trial #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>Height</td>
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</tbody>
</table>

Average height__________
Calculate $v_0$ for the launcher. This velocity will serve as your velocity in equation 7 for the next part of the lab. Show your work here below.

Considering equation 2, why does this equation give the correct initial velocity?
Part II: Range Measurements

Now you will launch the ball at various angles, and measure the range. Lay a strip of pressure sensitive paper along the surface of the table, in the direction that the ball will be launched. Using angles of 15, 30, 45, 60 and 75 degrees, you will launch the ball, and measure its range five times at each angle, and take the average for each angle. You may have to use a new strip, or move your strip, after completing your measurements for a given angle.

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
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</thead>
<tbody>
<tr>
<td>Trial 1 range</td>
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<td>Trial 2 range</td>
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<td>Trial 3 range</td>
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<td>Trial 4 range</td>
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<td>Trial 5 range</td>
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<td>Average range</td>
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<tr>
<td>Calculated range</td>
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<tr>
<td>Difference</td>
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</table>

Compare these ranges with those calculated with equation 7, using the \( v_0 \) measured in part I. Show a sample calculation here:

Do they agree? Do you see similar ranges for the 15 and 75 degree measurements? What about the 30 and 60 degree measurements? If you see a discrepancy, can you think of a reason that we have not accounted for, which would have an effect? Which angle gave you the greatest range?
Part III: Determining Launch Angle

At this point you are ready to apply your projectile motion knowledge toward accomplishing a goal. Your instructor will place a cup on the table, in the direction of the ball’s path. It is your job to pick a launch angle which will place the ball in the cup.

Remember that the cup has a height, so you shouldn’t use equation 7 to determine the launch angle. Using equations 1-4, come up with a means of calculating a launch angle for the ball. You are permitted to measure the distance from the launcher to the cup, but you can’t move the cup! **Show your work:**