1. 

\[ P = \frac{1}{\pi} \int_0^{0.1} e^{-x^2} \, dx \]

\[ = \frac{1}{\pi} \int_0^{0.1} \left(1 - x^2 + \frac{x^4}{2} - \ldots\right) \, dx \]

\[ = \frac{1}{\sqrt{\pi}} \left[ x - \frac{x^3}{3} + \frac{x^5}{10} \right]_0^{0.1} \]

\[ = \frac{1}{\sqrt{\pi}} \left[ 0.1 - \frac{.001}{3} + \frac{.00001}{10} \right] - \ldots \]

\[ = 0.5642 \left[ .09967 \right] \approx .0562 \]
2. \[ x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 4y = 0 \]

\[ y = A x^s \quad \frac{dy}{dx} = s A x^{s-1} \quad \frac{d^2 y}{dx^2} = s \cdot (s-1) A x^{s-2} \]

Substituting
\[ s(s-1) A x^s + s A x^s + 4 A x^s = 0 \]
\[ s^2 - 5s + 4 = 0 \quad s = -1 \quad s = 4 \]
\[ y = A_1 x^{-1} + A_2 x^4 \]

\[ x^{2i} = e^{2i \ln x} = \cos(2i \ln x) + i \sin(2i \ln x) \]
\[ x^{-2i} = e^{-2i \ln x} = \cos(-2i \ln x) - i \sin(-2i \ln x) \]

Redefining \( A_1 \) and \( A_2 \) or set
\[ y(x) = A \cos(2i \ln x) + B \sin(2i \ln x) \]
This is the series

\[
f(x) = 1 + \frac{4}{\pi} \sum_{n=1, \text{odd}}^{\infty} \left[ \frac{\sin \pi n x}{n^2} + \frac{1}{3} \sin \frac{2\pi n x}{3} + \frac{1}{5} \sin \frac{3\pi n x}{5} \right]
\]

Note that if you use formulas, you must scale the \( \sin \) and \( \cos \) functions.

\[
a_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} f(x) \cos \left( \frac{2\pi n x}{\ell} \right) dx \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx
\]

\[
b_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} f(x) \sin \left( \frac{2\pi n x}{\ell} \right) dx
\]

This function is neither even nor odd as it stands, but if you subtract out the average value \( a_0 = 1 \), then the remainder is odd.
4. \[ du = dq - P\, dV \Rightarrow du = -P\, dV \quad \#1 \]

\[ P\, dV + V\, dP = R\, dT \quad \#2 \]

\[ du = \frac{3}{2} R\, dT \quad \#3 \]

From \#1 + \#3

\[ -P\, dV = \frac{3}{2} R\, dT - \frac{3}{3} R\, dN = R\, dT \quad \#4 \]

From \#4 + \#2

\[ P\, dV + V\, dP = -\frac{2}{3} P\, dV \]

\[ \frac{5}{3} P\, dV = -V\, dP \]

\[ \frac{5}{3} \frac{dV}{V} = -\frac{dP}{P} \]

\[ \frac{5}{3} \ln V = -\ln P + \ln C \]

\[ \ln V^{\frac{5}{3}} = -\ln \frac{C}{P} \]

\[ V^{\frac{5}{3}} = \frac{C}{P} \Rightarrow PV^{\frac{5}{3}} = C \]

* To see this, think of \( T \) as a function of \( P + V \)

\[ T = \frac{1}{R} PV \quad \Delta T = \left( \frac{\partial T}{\partial P} \right)_V \Delta P + \left( \frac{\partial T}{\partial V} \right)_P \Delta V = \frac{1}{R} V\, dP + \frac{1}{R} P\, dV \]

\[ \therefore R\, dT = V\, dP + P\, dV \text{ or } R\, dT = V\, dP + P\, dV \]

Alternatively, you can differentiate implicitly.
\[ \Delta T = \frac{\partial T}{\partial l} \Delta l + \frac{\partial T}{\partial g} \Delta g = \frac{8\pi}{3} \frac{1}{2} \left( \frac{1}{2} \right) l^2 g^{-\frac{3}{2}} \Delta l + \frac{2\pi}{3} \left( -\frac{1}{2} \right) \frac{1}{2} l^2 g^{-\frac{3}{2}} \Delta g \]

\[ \Delta T = \frac{2\pi}{3} \frac{1}{2} l^2 g^{-\frac{3}{2}} \Delta l + \frac{2\pi}{3} \left( -\frac{1}{2} \right) l^2 g^{-\frac{3}{2}} \Delta g \]

\[ \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l} - \frac{1}{2} \frac{\Delta g}{g} \]

\[ \left( \frac{\Delta T}{T} \right)^2 = \frac{1}{4} \left( \frac{\Delta l}{l} \right)^2 + \left( \frac{\Delta g}{g} \right)^2 \]

\[ \frac{\Delta T}{T} = \frac{1}{2} \sqrt{\left( \frac{\Delta l}{l} \right)^2 + \left( \frac{\Delta g}{g} \right)^2} \]

\[ = \frac{1}{2} \sqrt{\left( \frac{0.001}{0.998} \right)^2 + \left( \frac{0.001}{0.980} \right)^2} \]

\[ = \frac{1}{2} \sqrt{1.01415 \times 10^{-8} + 1.04123 \times 10^{-8}} \]

\[ = \frac{1}{2} \times 0.300143 = 7.17 \times 10^{-5} \]

\[ \Delta T = 3.154 \times 10^{-7} \times 7.17 \times 10^{-5} = 2.261 \text{ seconds} \]

\[ \sim 37 \text{ minutes } 41 \text{ sec} \]

Alternatively, if \[ T = 2\pi \frac{1}{g} \ln T = 2\pi \frac{1}{g} + \frac{1}{2} \ln l - \frac{1}{2} \ln g \]

\[ \frac{\Delta T}{T} = \frac{1}{2} \frac{\Delta l}{l} - \frac{1}{2} \frac{\Delta g}{g} \]

\[ \left( \frac{\Delta T}{T} \right)^2 = \frac{1}{4} \left( \frac{\Delta l}{l} \right)^2 + \frac{1}{4} \left( \frac{\Delta g}{g} \right)^2 \]

\[ \rightarrow \text{ result given when the error term goes to zero statistically} \]
6. \[ X_{cm} = \frac{\int_{\theta_1}^{\theta_2} r \cos \theta \, dr \, d\theta}{M} \]

\[ X_{cm} = \rho \int_{\theta_1}^{\theta_2} \cos \theta \, d\theta \int_{r_1}^{r_2} r \, dr \]

\[ = \rho \sin \theta \left[ \frac{r^3}{3} \right]_{r_1}^{r_2} = \frac{1}{2} \sin \frac{\theta}{2} \frac{R^3}{3} \]

\[ = \frac{2}{3} \sin \frac{\theta_0}{2} \frac{R^3}{A} \]

Now \[ A = \left( \frac{\theta_0}{2\pi} \right) \pi R^2 = \frac{\theta_0 R^2}{2} \]

\[ X_{cm} = \frac{2}{3} \frac{\theta_0}{\theta_0} \frac{R^3}{2} = \frac{4}{3} \frac{\sin \frac{\theta_0}{2}}{\theta_0} R \]

Note that \[ X_{cm} \to 0 \text{ if } \theta_0 = 2\pi \]

\[ X_{cm} \to \frac{2R}{3} \text{ if } \theta_0 \to 0 \]

\[ \Rightarrow \frac{X_{cm}}{R} = \frac{2}{3} \left( 1 + \frac{\sin \frac{\theta_0}{2}}{\theta_0} \right) \]
7. \[ (1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} = 0 \]

One solution: \[ y_1 = C_1 \] where \( C_1 \) is any constant

To find 2nd solution let \( p = \frac{dy}{dx} \)

\[ (1-x^2) \frac{dp}{dx} = 2xp \]

\[ \int \frac{dp}{p} = \int \frac{2x \, dx}{1-x^2} \]

\[ \ln p = -\ln (1-x^2) + \ln C_2 \]

\[ p = \frac{C_2}{1-x^2} \]

\[ \frac{dy}{dx} = \frac{C_2}{1-x^2} \]

\[ y = C_2 \left( \int \frac{dx}{1-x^2} \right) + C_1 \]

\[ \frac{1}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x} \quad \frac{A(1-x) + B(1+x)}{1-x^2} = 1 \]

\[ A + B = 1 \quad -A + B = 0 \]

\[ A = B = \frac{1}{2} \]

\[ y_2 = \frac{C_2}{2} \left[ \int \frac{dx}{1-x} + \int \frac{dx}{1+x} \right] = \frac{C_2}{2} \left( \ln (1+x) - \ln (1-x) \right) \]

\[ y_2 = \frac{C_2}{2} \ln \left( \frac{1+x}{1-x} \right) = C_2' \ln \left( \frac{1+x}{1-x} \right) \]