Some Facts about Factorials.

By definition, \( n! = n(n-1)(n-2) \ldots (3)(2)(1) \). In words, the factorial of a number \( n \) is the product of \( n \) factors, starting with \( n \), then 1 less than \( n \), then 2 less than \( n \), and continuing on with each factor 1 less than the preceding one until you reach 1.

The conventional order of operations is for the factorial, as with other unary operators to have priority over binary operations of addition, subtraction, multiplication, and division. As a consequence, \( 3n! \) is not the same as \( (3n)! \). More precisely,

\[
3n! = (3n)(3n-1)(3n-2) \ldots (3)(2)(1), \quad \text{while} \quad (3n)! = (3n)(3n-1)(3n-2) \ldots (3)(2)(1).
\]

It's often helpful to note some of the terms not explicitly written in a factorial like \( (3n)! \), which can be rewritten:

\[
(3n)! = (3n)(3n-1)(3n-2) \ldots (2n+1)(2n)(2n-1) \ldots (n+1)(n)(n-1) \ldots (3)(2)(1).
\]

Noting some of the factors encountered "on the way down" in the factorial can help you see cancellation. For example,

\[
\frac{(3n)!}{(2n)!} = \frac{(3n)(3n-1)(3n-2) \ldots (2n+1)(2n)(2n-1) \ldots (n+1)(n)(n-1) \ldots (3)(2)(1)}{(2n)(2n-1) \ldots (n+1)(n)(n-1) \ldots (3)(2)(1)} = (3n)(3n-1)(3n-2) \ldots (2n+1)
\]

Exercises.

Simplify the ratios in #1-4 below:

1. \[
\frac{(n+1)!}{(n-1)!}
\]

2. \[
\frac{(n+1)!}{(2n-1)!}
\]

3. \[
\frac{(3n)!}{(2n)!n!}
\]

4. \[
\frac{(4n)!}{[(2n)!]^2}
\]

5. For each of the above four ratios, determine the limit, as \( n \to \infty \).