Differentiation Workshop
Math 101 – 01

Using the Differentiation Rules of Sections 2.3 and 2.4, differentiate, with respect to x, each of the functions of x in 1-5 below.

1. $\sin(x)\cos(x)$

2. $(x+\frac{1}{x})^2$

3. $\dfrac{2x}{x+1}$

4. $\dfrac{1+3x^3}{\sqrt{x}}$ (in 2 ways)

5. $(\cos(x))^2$ (more often expressed in streamlined form as $\cos^2 x$)

6. Write the **limit definition** of $D_x[f(x)g(x)]$.

7. To the expression $[f(x+h)g(x+h)]-[f(x)g(x)]$ add zero (Trick #1 in mathematics) in the form of $-f(x+h)g(x)+f(x+h)g(x)$. See if you can spot a way of associating the result so that when you divide the entire expression by h, you get two difference quotients.

Sample Solutions and Notes

1. $D_x(\sin(x)\cos(x)) = \sin(x)[-\sin(x)] + \cos(x)[\cos(x)] =$
$-[\sin(x)]^2 + [\cos(x)]^2 = \cos(2x)$

Note: The first step in the process above is the application of the Product Rule for Derivatives, the second is an algebraic simplification, and the final step is an application of a Double Angle Formula (Reference page $\boxed{2}$). Though it isn't essential to take that final step, it's good to be aware of such things as double angle formulas, so that you could refer to them (but not memorize them).

My Notes:

2. $(x+\frac{1}{x})^2 = x^2 + 2(x)(\frac{1}{x}) + (\frac{1}{x})^2 = x^2 + 2 + x^{-2}$, so

$D_x[(x+\frac{1}{x})^2] = D_x[x^2 + 2 + x^{-2}] = D_x[x^2] + D_x[2] + D_x[x^{-2}] =$

$2x + 0 + (-2)x^{-3} = 2x - \frac{2}{x^3}$

Note: The Power Rule doesn't directly apply, so the first step in the process above is to rewrite the expression, not immediately differentiate. It is rewritten so that the differentiation rules we have can be applied. Later on in the course, we'll derive a rule, called the Chain Rule, that will enable us to differentiate without first doing an algebraic rewrite. Again, it isn't essential to take the final step, but it's good to be aware that such an algebraic rewrite is possible.

My Notes:

3. $D_x[\frac{2x}{x+1}] = \frac{(x+1)(2) - 2x(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$

Note: This one is a direct application of the Quotient Rule.

My Notes:

4. $\dfrac{1+3x^3}{\sqrt{x}} = \dfrac{1}{\sqrt{x}} + \dfrac{3x^3}{\sqrt{x}} = x^{-1/2} + 3x^{5/2}$, so

$$D_x[\frac{1+3x^3}{\sqrt{x}}] = D_x[x^{-1/2} + 3x^{5/2}] = (-1/2)x^{-3/2} + (3)(5/2)x^{3/2} =$$

$$\frac{-1}{2\sqrt{x^3}} + \frac{15\sqrt{x^3}}{2}$$

and, alternatively,

$$D_x[\frac{1+3x^3}{\sqrt{x}}] = \frac{\sqrt{x}(9x^2) - (1+3x^3)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} =$$

$$\frac{9x^{5/2} - \frac{1}{2\sqrt{x}} - \frac{3x^{5/2}}{2}}{x} = \frac{\frac{15x^{5/2}}{2} - \frac{1}{2\sqrt{x}}}{x} = \frac{15x^{5/2}}{2x} - \frac{1}{2x\sqrt{x}} =$$

$$\frac{15x^{3/2}}{2} - \frac{1}{2x^{3/2}}$$

Note: This problem is designed, in part, as a review of rational expressions and exponents! It also shows the choice one often makes between rewriting the algebraic form prior to differentiating (the first method above) and directly differentiating (the alternate method, which applies the Quotient Rule immediately).

Question: Why don't we have a similar choice in #3?

My Notes:

5. $\cos^2 x = (\cos x)(\cos x)$, so

$$D_x \cos^2 x = D_x[(\cos x)(\cos x)] = (\cos x)(-\sin x) + (\cos x)(-\sin x) =$$
$$-2\sin x\cos x = -\sin(2x)$$

Note: Once again, the Power Rule doesn't apply, but, after rewriting the square of the cosine as a product, the Product Rule applies. The final step uses another Double Angle formula.

My Notes: