Math 324  Problem 11  Solution  9 March 2004

\[
\left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)
\]

\[
= V \rho + \rho \frac{\partial}{\partial x} \left( \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) \\
+ \rho \frac{\partial}{\partial y} \left( \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) \\
+ \rho \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right)
\]

\[
= V \frac{\partial \rho}{\partial x} + \rho \frac{\partial V}{\partial x} + \rho \frac{\partial V}{\partial y} + \rho \frac{\partial V}{\partial z}
\]

[using \( \frac{\partial V}{\partial x} \left( \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right) = 0 \), etc.]

\[
= \left( \frac{\partial}{\partial x} \rho V \right)
\]

Thus \( \int_V \rho \, dx \, dy \, dz = \frac{\partial}{\partial x} \left( \rho V \right) \)