Math 324  Problem 21  Solution  April 20, 2004

How many monomials in $n$ variables of degree $k$?

Let $n N_k$ be this number. If $n=1$ (i.e., just one variable $x$) there is only one monomial of degree $k$, namely $x^k$. That is, $1 N_k = 1$. If $n=2$ (i.e., there are two variables, $(x,y)$), then every monomial in $x$ of degree $\leq k$ can be brought up to degree $k$ by multiplying with the appropriate power of $y$: $y^k x$, $y^{k-1} x^2$, $y^{k-2} x^3$, ... , $x^k$.

That is, $2 N_k = \sum_{k'=0}^{k} N_{k'} = k+1$.

This observation gives a recursive formula for $n N_k$:

$$n N_k = \sum_{k'=0}^{k} n N_{k'}$$

in terms of $n-1 N_k$.

In particular, $3 N_k = \sum_{k'=0}^{k} (k'+1) = 1+2+...+(k+1) = \frac{(k+1)(k+2)}{2}$.