1. Find $f'(x)$. Do not simplify.

(a) $f(x) = \frac{x^2 + 1}{3x^3 - 5}$.

Solution: By the quotient rule, we get

$$f'(x) = \frac{(3x^3 - 5) \cdot (2x) - (x^2 + 1) \cdot (9x^2)}{(3x^3 - 5)^2}.$$ 

(b) $f(x) = \cos(\sqrt{1 - x^2})$

Solution: We get

$$f'(x) = -\sin(\sqrt{1 - x^2}) \cdot \frac{1}{2} (1 - x^2)^{-\frac{1}{2}} \cdot (-2x).$$
2. Suppose $F$, $f$, and $g$ are functions and that $F = f \circ g$. Use the information given in the following table of values to find an equation for the line tangent to the curve $y = F(x)$ at $x = 1$.

<table>
<thead>
<tr>
<th></th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
<th>$g(x)$</th>
<th>$g'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Solution: First, we have $F(1) = f(g(1)) = f(3) = 2$, so our line passes through the point $(1, 2)$. Next, to find the slope, we use the fact that

$$F'(1) = f'(g(1)) \cdot g'(1)$$
$$= f'(3) \cdot 2$$
$$= (-2) \cdot 2$$
$$= -4.$$

Using the point-slope form for the equation of a line, we get

$$y - 2 = -4(x - 1).$$
3. Find an equation for the line tangent to the curve \( y = \sin(2x) \) at the point where \( x = \frac{\pi}{3} \). Leave your answer in exact form (in terms of \( \pi \) and square roots).

Solution: We have

\[
y\left(\frac{\pi}{6}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}.
\]

To find the slope, we take

\[
y' = 2\cos(2x)
\]

and evaluate at \( x = \frac{\pi}{3} \). We get

\[
y'\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{2\pi}{3}\right) = -1.
\]

The equation for the line is

\[
\left(y - \frac{\sqrt{3}}{2}\right) = -\left(x - \frac{\pi}{3}\right).
\]
4. Find the slope of the line tangent to the curve $x^3 + 2y^3 - x^2y^4 = x$ at the point $(2, -1)$.

Solution: The slope is $\frac{dy}{dx}$, which we find by implicit differentiation. We get

$$3x^2 + 6y^2 y' - 4x^2 y^3 y' - 2xy^4 = 1,$$

which we solve to get

$$y' = \frac{1 + 2xy^4 - 3x^2}{6y^2 - 4x^2 y^3}.$$

At the point $x = 2, y = -1$, we have

$$y' = \frac{1 + 4 - 12}{6 + 16} = \frac{-7}{22}.$$
5. A particle moves along a straight line with position \( x \) (in inches) as a function of \( t \) (in seconds) given by

\[
x(t) = t^3 - 6t^2 + 9t.
\]

(a) Give the particle’s direction of motion and its velocity at time \( t = 2 \). Be sure to include the proper units.

Solution: We have

\[
x'(t) = 3t^2 - 12t + 9.
\]

The velocity at \( t = 2 \) is \( x'(2) = 3 \times 4 - 12 \times 2 + 9 = -3 \). The particle is moving in the negative direction at 3 inches per second.

(b) At what times does the particle change direction?

Solution: The particle changes direction when its velocity changes sign. Since the velocity is a continuous function, this can happen only when the velocity is zero. We have

\[
x'(t) = 3t^2 - 12t + 9
= 3(t^2 - 4t + 3)
= 3(t - 1)(t - 3),
\]

which changes sign when \( t = 1 \) and when \( t = 3 \).
6. The marketing department at Amalgamated Widget finds that the cost $C$ (in dollars) of producing $x$ cartons of squash balls given by

$$C(x) = 3000 + 12x - 0.01x^2.$$ 

Find and interpret the marginal cost at a production level of 280 cartons of squash balls.

Solution: We have

$$C'(x) = 12 - 0.02x$$

so that the marginal cost at $x = 280$ is given by

$$C'(280) = 12 - 0.02 \times 280 = 6.40.$$ 

When the production level is 280 cartons of squash balls, the additional cost of producing another carton is approximately $6.40.$
7. Concrete is being poured into a conical tank (vertex down) at the rate of 2 cubic feet per second. The tank is 10 feet deep and has a radius at the top of 5 feet. At what rate is the level of concrete in the tank rising when it has been filled to a depth of 6 feet? (The volume of a cone with radius \( r \) and height \( h \) is given by \( \frac{1}{3} \pi r^2 h \)).

Solution: As in the drawing, let \( x \) and \( y \) denote the depth and radius of the already-poured concrete. By similar triangles, we find that \( x = \frac{y}{2} \), so we can express the volume \( V \) of the already-poured concrete as

\[
V = \frac{1}{3} \pi \left( \frac{y}{2} \right)^2 y = \frac{\pi}{12} y^3.
\]

Differentiating both sides with respect to \( t \), we find that

\[
\frac{dV}{dt} = \frac{3\pi}{12} y^2 \frac{dy}{dt} = \frac{\pi y^2}{4} \frac{dy}{dt}.
\]

When \( y = 6 \), the equation becomes

\[
\frac{dV}{dt} = 9\pi \frac{dy}{dt}.
\]

We are given that \( \frac{dV}{dt} = 2 \), so we can solve for \( \frac{dy}{dt} \) (which is what we want). We get

\[
\frac{dy}{dt} = \frac{2}{9\pi} \text{ feet per second}.
\]