Math 101(04) Practice Hour Exam #1 25 September 2001

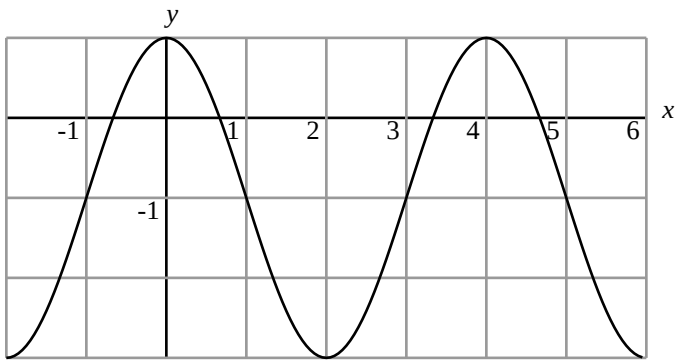
1. A photograph of a certain tree taken on January 1, 1980 shows that the tree was 12 ft tall on that date. On January 1, 1995, another photograph shows that the same tree was 37 ft tall.

   (a) Assuming a linear model is correct, find D, the current date (measured in years CE), as a function of h, the height of the tree.

   (b) What is the slope in this model? What is the meaning of the slope in terms of the tree?

   (c) What is the D-intercept in this model? What is the meaning of the D-intercept in terms of the tree?

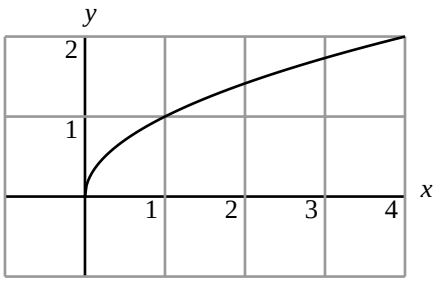
2. The diagram shows the graph of a function f given by $f(x) = A\sin(B(x+C)) + D$. Find possible values for A, B, C, and D.

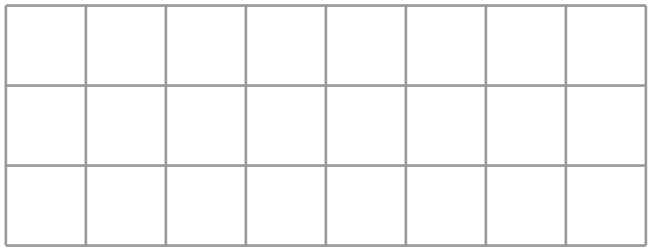


3. The graph on the left shows the curve $y = \sqrt{x}$. In the grid on the right, plot the graph of a function f given by

$$f(x) \quad = \quad -\sqrt{\frac{1}{2}(x+2)}.$$

Explain how to do this without using a graphing calculator or playing "connect-the-dots."





4. Let $f(x) = x^2 - 2x$ and $g(x) = \sqrt{3x+2}$.

   (a) Find a formula for $f \circ g(x)$.

   (b) Find a formula for $f \circ f(x)$. Simplify as far as possible.

5. Find the indicated limits.

   (a) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$.

   (b) $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$.

   (c) $\lim_{x \to 3^-} \dfrac{x - 5}{(x-3)(x-1)}$.

   (d) $\lim_{x \to 2} x^2 + 5x + 1$.

6. Let $f(x) = 4x - x^2$.

   (a) Let $m(a)$ denote the slope of the secant line through the points $(2, f(2))$ and $(a, f(a))$. Find a formula for $m(a)$. Simplify as far as possible.

   (b) Find a value of a so that the slope of the secant line through $(2, f(2))$ and $(a, f(a))$ is 5.

7. Let f be the function given by

$$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ 1-(x-1)^2 & \text{if } 0 \le x < 1 \\ 0 & \text{if } x = 1 \\ 2-x & \text{if } 1 < x \le 2 \\ \sin(x-2) & \text{if } x > 2 \end{cases}$$

   At each point, indicate whether f is continuous, has a removable discontinuity, has a jump discontinuity, or has an infinite discontinuity. Give reasons.

   (a) $x = 0$

   (b) $x = 1$

   (c) $x = 2$

8. Use the Intermediate Value Theorem to prove that there is a number c between 0 and $\dfrac{\pi}{2}$ satisfying the equation

$$c = \cos c.$$