Math 101(04) **Practice Exam IV** **26 November 2001**

1. Let f be the function given by $f(x) = x^4 + x^3 - 5x^2 + 6$.

   (a) List the intervals on which f is increasing and the intervals on which f is decreasing.

   (b) List the values of x at which f has a local maximum and the values of x at which f has a local minimum.

   (c) List the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward.

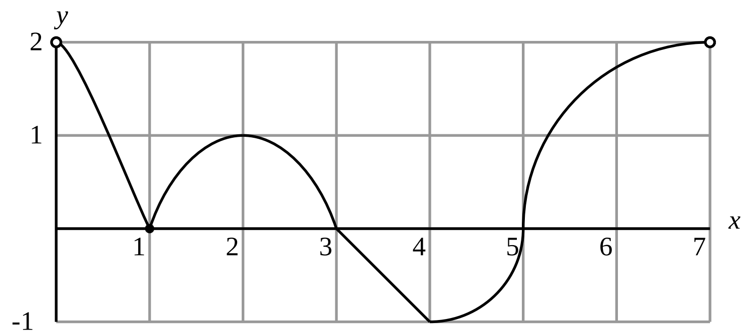
2. Let $f(x) = \dfrac{x\sqrt{3x^2+5}}{(2x+1)^2}$.

(a) Find $\lim_{x \to \infty} f(x)$.

(b) Find $\lim_{x \to -\infty} f(x)$.

(c) Find $\lim_{x \to -\frac{1}{2}^+} f(x)$.

3. The function f is continuous and differentiable on the interval $[0, 6]$. Here is a graph of the *derivative* of f. That is, this graph shows the curve $y = f'(x)$.



(a) List the values of x at which f has a local maximum and the values of x at which f has a local minimum.

(b) On what intervals is the graph of f concave upward? On what intervals is the graph of f concave downward?

4. The equation $x^3 + 4x + 4 = 0$ has a single root, near $x = -1$.

(a) Set up the iteration rule you would use in applying Newton's method to find the root of the equation above. Your rule should give a formula for x_{n+1} in terms of x_n.

$$x_{n+1} = \boxed{}$$

(b) Using the initial guess $x_0 = -1$, find x_1. Leave your answer in exact form.

5. Find the positive number x for which $5x + \dfrac{1}{x^2}$ is as small as possible.

6. One corner of a rectangle is at the origin in the xy-plane, and the opposite corner lies in the first quadrant, along the line $y = 7 - \frac{2}{3}x$. The sides of the rectangle are parallel to the coordinate axes. Find the largest possible area of such a rectangle.