1. Let $f$ be the function given by $f(x) = x^4 + x^3 - 5x^2 + 6$.

(a) List the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing.

(b) List the values of $x$ at which $f$ has a local maximum and the values of $x$ at which $f$ has a local minimum.

(c) List the intervals on which the graph of $f$ is concave upward and the intervals on which the graph of $f$ is concave downward.
2. Let \( f(x) = \frac{x\sqrt{3x^2 + 5}}{(2x + 1)^2} \).

(a) Find \( \lim_{x \to \infty} f(x) \).

(b) Find \( \lim_{x \to -\infty} f(x) \).

(c) Find \( \lim_{x \to \frac{1}{2}^+} f(x) \).
3. The function $f$ is continuous and differentiable on the interval $[0, 6]$. Here is a graph of the derivative of $f$. That is, this graph shows the curve $y = f'(x)$.

(a) List the values of $x$ at which $f$ has a local maximum and the values of $x$ at which $f$ has a local minimum.

(b) On what intervals is the graph of $f$ concave upward? On what intervals is the graph of $f$ concave downward?
4. The equation $x^3 + 4x + 4 = 0$ has a single root, near $x = -1$.

(a) Set up the iteration rule you would use in applying Newton’s method to find the root of the equation above. Your rule should give a formula for $x_{n+1}$ in terms of $x_n$.

$x_{n+1} =$

(b) Using the initial guess $x_0 = -1$, find $x_1$. Leave your answer in exact form.
5. Find the positive number $x$ for which $5x + \frac{1}{x^2}$ is as small as possible.
6. One corner of a rectangle is at the origin in the $xy$-plane, and the opposite corner lies in the first quadrant, along the line $y = 7 - \frac{2}{3}x$. The sides of the rectangle are parallel to the coordinate axes. Find the largest possible area of such a rectangle.